筑波大学
University of Tsukuba

IMAGINE THE FUTURE.

ベイズ統計と構造力学
Bayesian and Structure Mechanics

フロンティア工学研究グループ 01
LAFEE

システム情報系 助教
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Bayes Theorem

likelihood
尤度

prior distribution
事前分布

$$p(\boldsymbol{H}|\boldsymbol{D}) = \frac{p(\boldsymbol{D}|\boldsymbol{H}) \times p(\boldsymbol{H})}{p(\boldsymbol{D})}$$

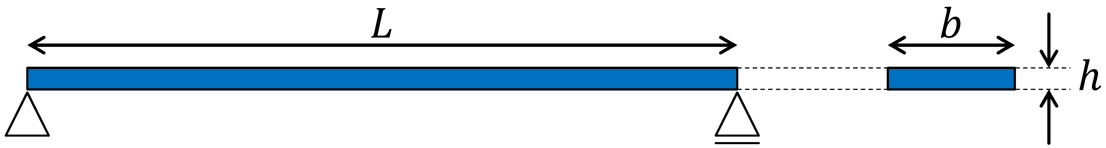
posterior distribution
事後分布

$\boldsymbol{H}$: Parameters
$\boldsymbol{D}$: Data

- If we know the relationship between "What we can observe ($\boldsymbol{D}$)" and "What we'd like to know ($\boldsymbol{H}$)", we can estimate "What we'd like to know ($\boldsymbol{H}$)" from "What we can observe ($\boldsymbol{D}$)" by Bayes Theorem
  - もし、「$\boldsymbol{D}$：観測しやすいもの」と「$\boldsymbol{H}$：知りたいもの」の関係が分かっているならば、ベイズ理論により「$\boldsymbol{D}$：観測しやすいもの」から「$\boldsymbol{H}$：知りたいもの」を推定できる

Example

- Estimate "**Young Modulus"** of a Beam from observed natural frequency.



- **Material Parameters** (Young Modulus: E and Density: ρ) follow logarithm normal distributions.

- **Member Parameters** (Length: L, Width: b, Height: h) follow logarithm normal distributions.

- **Observation Noise** into the Natural Frequency follows a normal distribution.

Simulating Data

I	A
5.08E-10	0.001508

- In this time, data is **simulated** instead of **experiment**.
  - Let's generate Observed Natural Frequency: f by **Monte Carlo** Simulation
  - **Setting** Statistical characteristics of Parameters

	Material		Member		
	E	ρ	b	h	L
	[N/m^2]	[kg/m^3]	[m]	[m]	[m]
Average	2.05×10^{11}	7850	0.7501	0.00201	1.829
S.D.	2.00×10^{9}	5	0.0015	0.00001	0.001

	Member	
	I	A
	[m]	[m]
Average	5.08E-10	0.001508

Basis of Statistics

- X follows Logarithm Normal Distribution
  - X's Average: μ_X, Standard Deviation: σ_X

$$p(X) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\ln X - \mu)^2}{2\sigma^2}\right]$$

Average and Standard Deviation

$$\mu_X = \exp\left[\mu + \frac{\sigma^2}{2}\right]$$

$$\sigma_X = \exp[2\mu + \sigma^2]\,(\exp\sigma^2 - 1)$$

Parameters of Log. Norm. Dist.

$$\sigma = \sqrt{\ln\left[1 - \left(\frac{\sigma_X}{\mu_X}\right)^2\right]}$$

$$\mu = \ln\mu_X - \frac{\sigma^2}{2}$$

	E [N/m^2]
Average	2.05×10^{11}
S.D.	2.00×10^{9}

	E [N/m^2]
μ	26.046
σ	0.010

Monte Carlo Simulation

☐ Generate X

Frequency

Sample	E	ρ	I	A	b	h	L	f
1	2.10.E+11	7847.731	5.087E-10	1.509.E-03	7.502.E-01	2.011.E-03	1.829414	1.41
2	2.07.E+11	7856.437	5.151E-10	1.514.E-03	7.492.E-01	2.021.E-03	1.828602	1.41
3	2.04.E+11	7852.566	5.103E-10	1.515.E-03	7.536.E-01	2.010.E-03	1.828302	1.39
4	2.08.E+11	7846.214	4.969E-10	1.498.E-03	7.509.E-01	1.995.E-03	1.828626	1.39
5	2.08.E+11	7850.578	5.101E-10	1.511.E-03	7.504.E-01	2.013.E-03	1.829197	1.40
6	2.06.E+11	7852.279	5.114E-10	1.513.E-03	7.510.E-01	2.014.E-03	1.830040	1.40
7	2.05.E+11	7840.501	5.08E-10	1.509.E-03	7.509.E-01	2.010.E-03	1.827074	1.40
8	2.08.E+11	7851.889	5.013E-10	1.499.E-03	7.483.E-01	2.003.E-03	1.830499	1.40
9	2.05.E+11	7846.629	5.015E-10	1.501.E-03	7.498.E-01	2.002.E-03	1.829832	1.39
10	2.07.E+11	7853.577	5.083E-10	1.508.E-03	7.497.E-01	2.011.E-03	1.827193	1.40

☐ Natural Frequency

$$f = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}}$$

Observing Data

- Simulating Observed Data including Noise

Freq.	Noise	Observed Freq.
1.41	-0.000575	**1.408**
1.41	-0.000463	**1.407**
1.39	-0.001111	**1.390**
1.39	-0.001155	**1.391**
1.40	-0.000788	**1.404**
1.40	-0.000414	**1.396**
1.40	0.000034	**1.395**
1.40	-0.002033	**1.394**
1.39	-0.000094	**1.386**
1.40	-0.000160	**1.402**

Bayes Theorem

- Bayes Theorem

$$p(E|f) = \frac{p(f|E) \times p(E)}{p(f)}$$

- Likelihood:

$$p(f|E) = \frac{1}{\sqrt{2\pi}s} \exp\left[-\frac{\left(f - \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}}\right)^2}{2s^2}\right]$$

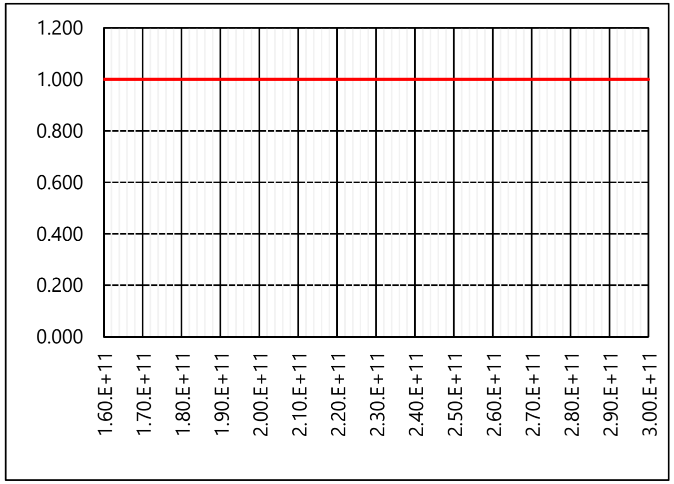
  - If $f = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}}$, Likelihood: $p(f|E)$ returns the maximum value.
  - The variance: s^2 can be set freely

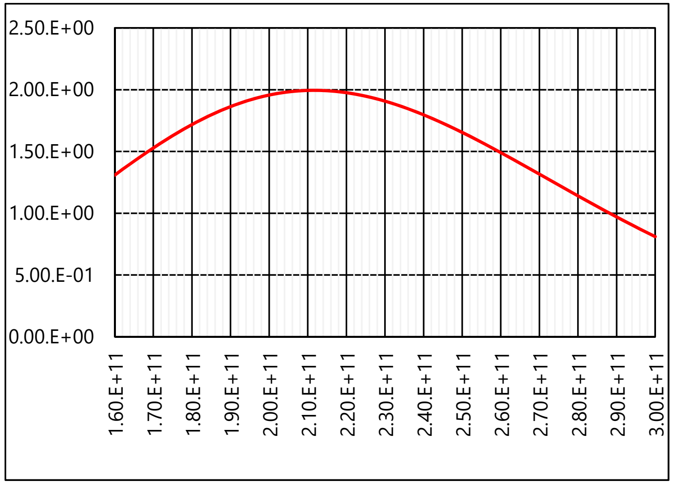
Estimation of Parameter

- Step One
  - From the Prior Distribution, we can estimate the distribution

$$\frac{p(f|E) \times p(E)}{p(f)} = p(E|f)$$

First Observed data:
$f = 1.408$





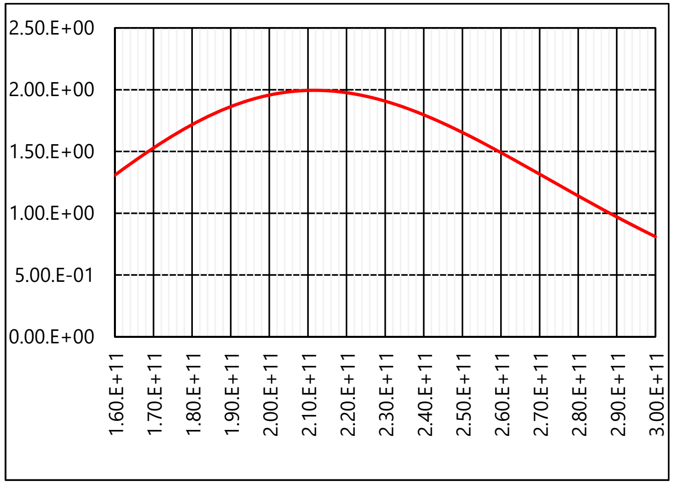
Estimation of Parameter

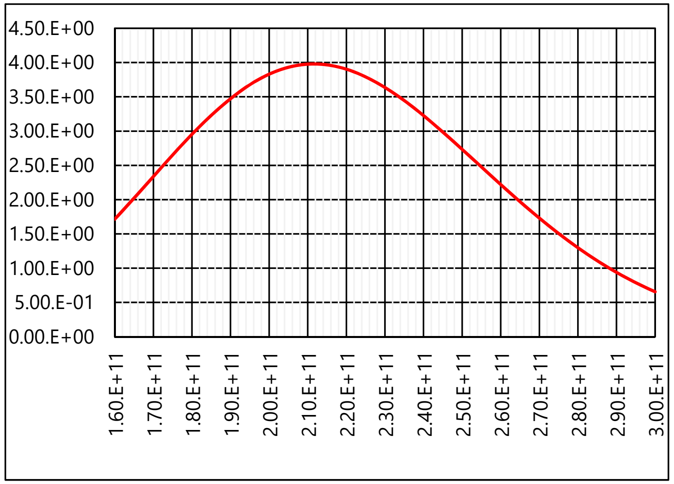
- Step: 2
  - From the Prior Distribution, we can estimate the distribution

The last **posterior** dist.
is taken as the new **prior** dist

$$\frac{p(f|E) \times p(E)}{p(f)} = p(E|f)$$

2nd Observed data:
$f = 1.407$

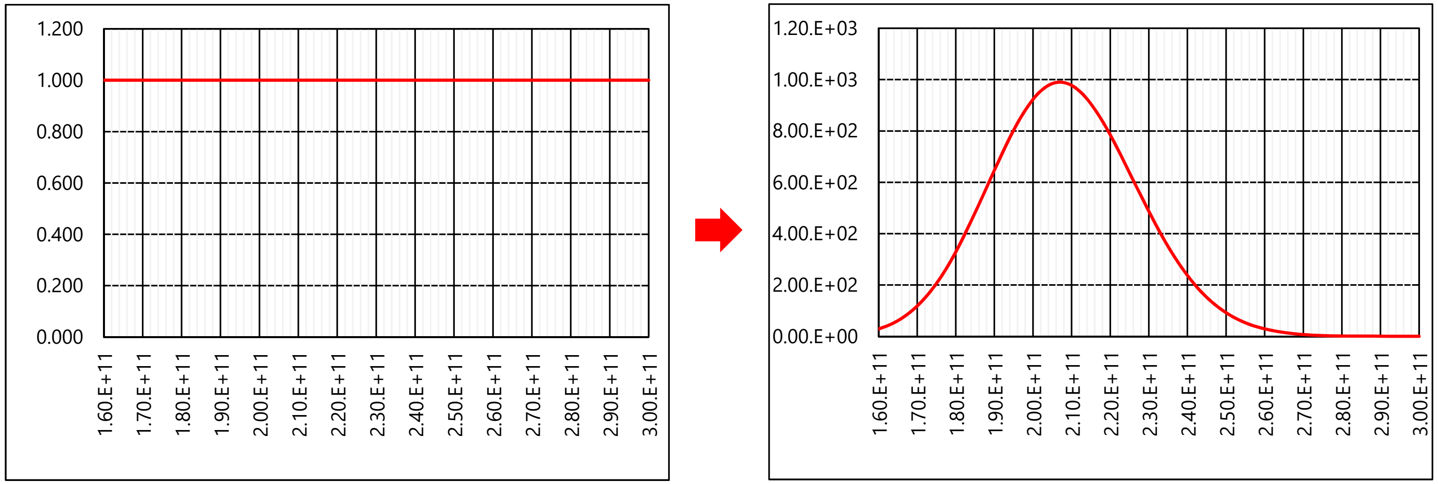




Estimation of Parameter

- Repeat the procedure
  - By the **production** of Likelihoods into which data: f are substituted

$$p(E|f) = \frac{p(f_n|E) \times \cdots \times p(f_2|E) \times p(f_1|E) \times p(E)}{C}$$



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