2020/11/24

固体の大変形問題

有限要素法の適用と応用

モノが壊れる現象を
シミュレートしたい

モノが
壊れる

＝

モノの
大変形

シミュレーションには
有限要素法を用いる。

力のつりあい式

$$\int_S \boldsymbol{t}\,\mathrm{d}a + \int_B \boldsymbol{b}\,\mathrm{d}v = \boldsymbol{0}$$

$\times$

有限要素法

$$\int_B \boldsymbol{w}\cdot\boldsymbol{r}\,\mathrm{d}v = 0$$

大変形の力のつりあい式

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \begin{bmatrix} & \mathbf{B} & \end{bmatrix} \begin{Bmatrix} \boldsymbol{s} \end{Bmatrix} \mathrm{d}V$$

大変形の力のつりあい式

$$
\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}
\end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V
$$

大変形の力のつりあい式

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\begin{array}{ccc|ccc} \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\ \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\ \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \end{array}\right] \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ \hdashline s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V$$

大変形の力のつりあい式

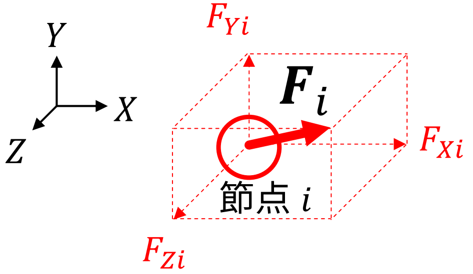
自由度数

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \begin{bmatrix} \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\ \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\ \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V$$

$$\boldsymbol{F}_X=\begin{Bmatrix}\vdots\\ F_{Xi}\\ \vdots\end{Bmatrix}\qquad \boldsymbol{F}_Y=\begin{Bmatrix}\vdots\\ F_{Yi}\\ \vdots\end{Bmatrix}\qquad \boldsymbol{F}_Z=\begin{Bmatrix}\vdots\\ F_{Zi}\\ \vdots\end{Bmatrix}$$

$$\begin{Bmatrix}\boldsymbol{F}_X\\ \boldsymbol{F}_Y\\ \boldsymbol{F}_Z\end{Bmatrix}=\int_B\begin{bmatrix}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}\\ \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}\\ \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}\end{bmatrix}\begin{Bmatrix}S_{11}\\ S_{22}\\ S_{33}\\ S_{12}\\ S_{23}\\ S_{31}\end{Bmatrix}\mathrm{d}V$$

各節点の
外力ベクトル



$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{Bmatrix} \Rightarrow \begin{Bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{12}\\ \tau_{23}\\ \tau_{31}\end{Bmatrix} \Rightarrow \begin{Bmatrix}\pi_{11}\\ \pi_{22}\\ \pi_{33}\\ \pi_{12}\\ \pi_{21}\\ \pi_{23}\\ \pi_{32}\\ \pi_{31}\\ \pi_{13}\end{Bmatrix} \Rightarrow \begin{Bmatrix}s_{11}\\ s_{22}\\ s_{33}\\ s_{12}\\ s_{23}\\ s_{31}\end{Bmatrix}$$

Cauchy応力 **(真応力)**

Kirchhoff応力

第1Piola-Kirchhoff応力 **（公称応力）**

第2Piola-Kirchhoff応力 **（疑似応力）**

$$\begin{Bmatrix}\boldsymbol{F}_X\\ \boldsymbol{F}_Y\\ \boldsymbol{F}_Z\end{Bmatrix} = \int_B \begin{bmatrix} \frac{\partial N}{\partial X}+\frac{\partial u}{\partial X}\frac{\partial N}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial N}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial N}{\partial Z} & \frac{\partial N}{\partial Y}+\frac{\partial u}{\partial X}\frac{\partial N}{\partial Y}+\frac{\partial u}{\partial Y}\frac{\partial N}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial N}{\partial Z}+\frac{\partial u}{\partial Z}\frac{\partial N}{\partial Y} & \frac{\partial N}{\partial Z}+\frac{\partial u}{\partial Z}\frac{\partial N}{\partial X}+\frac{\partial u}{\partial X}\frac{\partial N}{\partial Z} \\ \frac{\partial v}{\partial X}\frac{\partial N}{\partial X} & \frac{\partial N}{\partial Y}+\frac{\partial v}{\partial Y}\frac{\partial N}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial N}{\partial Z} & \frac{\partial N}{\partial X}+\frac{\partial v}{\partial X}\frac{\partial N}{\partial Y}+\frac{\partial v}{\partial Y}\frac{\partial N}{\partial X} & \frac{\partial N}{\partial Z}+\frac{\partial v}{\partial Y}\frac{\partial N}{\partial Z}+\frac{\partial v}{\partial Z}\frac{\partial N}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial N}{\partial X}+\frac{\partial v}{\partial X}\frac{\partial N}{\partial Z} \\ \frac{\partial w}{\partial X}\frac{\partial N}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial N}{\partial Y} & \frac{\partial N}{\partial Z}+\frac{\partial w}{\partial Z}\frac{\partial N}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial N}{\partial Y}+\frac{\partial w}{\partial Y}\frac{\partial N}{\partial X} & \frac{\partial N}{\partial Y}+\frac{\partial w}{\partial Y}\frac{\partial N}{\partial Z}+\frac{\partial w}{\partial Z}\frac{\partial N}{\partial Y} & \frac{\partial N}{\partial X}+\frac{\partial w}{\partial Z}\frac{\partial N}{\partial X}+\frac{\partial w}{\partial X}\frac{\partial N}{\partial Z} \end{bmatrix} \begin{Bmatrix}s_{11}\\ s_{22}\\ s_{33}\\ s_{12}\\ s_{23}\\ s_{31}\end{Bmatrix} \mathrm{d}V$$

各積分点の
応力ベクトル

$$
\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\begin{array}{ccc:ccc}
\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}
\end{array} \right] \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ \hdashline s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V
$$

各積分点の
Bマトリクス

線形Bマトリクス
（微小変形問題と共通）

$$
\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}
\end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V
$$

各積分点の
Bマトリクス

線形Bマトリクス
（微小変形問題と共通）

＋

非線形Bマトリクス
（大変形問題のみ）

$$
\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\begin{array}{ccc|ccc}
\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}
\end{array} \right] \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V
$$

各積分点の
Bマトリクス

まとめると

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\quad \mathbf{B} \quad \right] \begin{Bmatrix} \boldsymbol{s} \end{Bmatrix} \mathrm{d}V$$

モノが壊れる問題を解く。 ＝

$$\begin{Bmatrix} F_X \\ F_Y \\ F_Z \end{Bmatrix} = \int_B \begin{bmatrix} & \mathbf{B} & \end{bmatrix} \begin{Bmatrix} \boldsymbol{s} \end{Bmatrix} \mathrm{d}V$$

を解く

たとえば、構成則を代入すれば解ける

$$\boxed{\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \begin{bmatrix} & \mathbf{B} & \end{bmatrix} \begin{Bmatrix} \boldsymbol{s} \end{Bmatrix} \mathrm{d}V} = \int_B \begin{bmatrix} & \mathbf{B} & \end{bmatrix} \begin{bmatrix} & \mathbf{D} & \end{bmatrix} \begin{Bmatrix} \boldsymbol{E} \end{Bmatrix} \mathrm{d}V$$

$$\boxed{\begin{Bmatrix} \boldsymbol{s} \end{Bmatrix} = \begin{bmatrix} & \mathbf{D} & \end{bmatrix} \begin{Bmatrix} \boldsymbol{E} \end{Bmatrix}}$$

たとえば、歪ー変形関係式を代入すれば解ける

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\ \mathbf{B}\ \right]\left[\ \mathbf{D}\ \right]\left\{\boldsymbol{E}\right\} \mathrm{d}V = \int_B \left[\ \mathbf{B}\ \right]\left[\ \mathbf{D}\ \right]\left[\ \mathbf{B}^{\mathrm{T}}\ \right] \mathrm{d}V \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

$$\left\{\boldsymbol{E}\right\} = \left[\ \mathbf{B}^{\mathrm{T}}\ \right] \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

つまり

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\quad \mathbf{B} \quad \right] \left[\quad \mathbf{D} \quad \right] \left[\quad \mathbf{B}^{\mathrm{T}} \right] \mathrm{d}V \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

を解く

まとめると

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

とできる

簡単でしょ？

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

しかし・・・

そもそも、この式は
どこからきた？

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\quad \mathbf{B} \quad \right] \left\{ \boldsymbol{s} \right\} \mathrm{d}V$$

そもそも、この式は どこからきた？

$$
\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \left[\begin{array}{ccc:ccc}
\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\
\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}
\end{array}\right]
\left\{\begin{array}{c} s_{11} \\ s_{22} \\ s_{33} \\ \hdashline s_{12} \\ s_{23} \\ s_{31} \end{array}\right\} \mathrm{d}V
$$

出発点は
力のつりあい式

$$\int_S \boldsymbol{t}\,\mathrm{d}a + \int_B \boldsymbol{b}\,\mathrm{d}v = \mathbf{0}$$

と、

変形勾配テンソル

$$\mathrm{d}\boldsymbol{x} = \mathbf{F}\mathrm{d}\boldsymbol{X}$$

力のつりあい式

$$\int_S \boldsymbol{t}\,\mathrm{d}a + \int_B \boldsymbol{b}\,\mathrm{d}v = \boldsymbol{0}$$

×

変形勾配テンソル

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{F}\mathrm{d}\boldsymbol{X}$$

トラクション（表面力）　　体積力

$$\int_S \boldsymbol{t}\, da + \int_B \boldsymbol{b}\, dv = \boldsymbol{0}$$

ゼロ・ベクトル

トラクション（表面力）

体積力

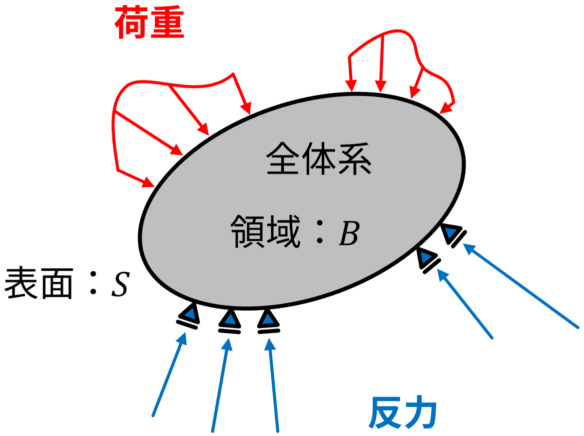
$$\int_S \boldsymbol{t}\, da + \int_B \boldsymbol{b}\, dv = \boldsymbol{0}$$

ゼロ・ベクトル

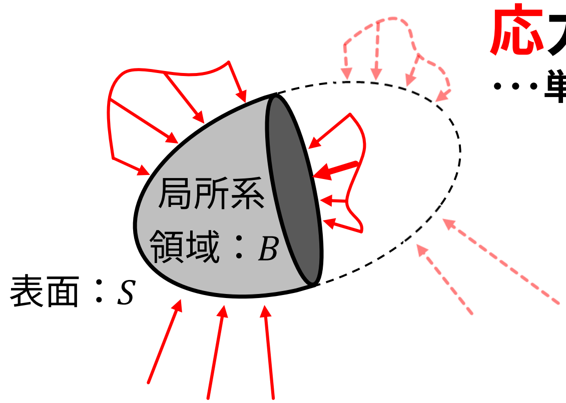
トラクション（表面力）

$$\int_S \boldsymbol{t}\, da + \int_B \boldsymbol{b}\, dv = \boldsymbol{0}$$

体積力　ゼロ・ベクトル



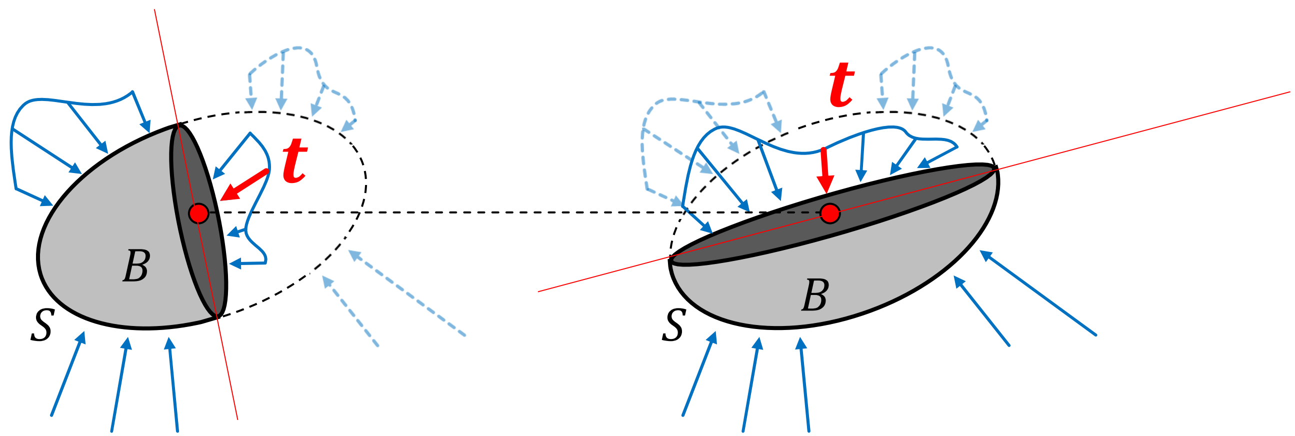
全体系に働く力･･･**外力**



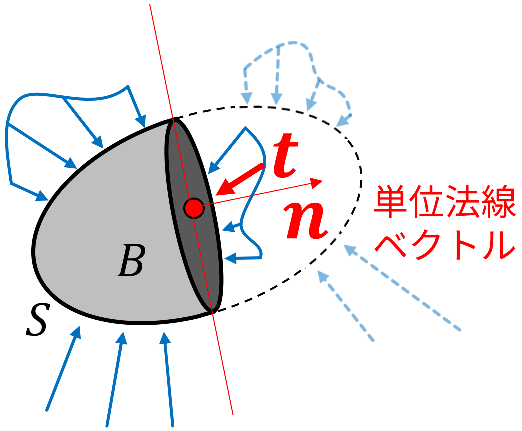
応力ベクトル
･･･単位面積あたりの内力

局所系に働く力･･･**内力**

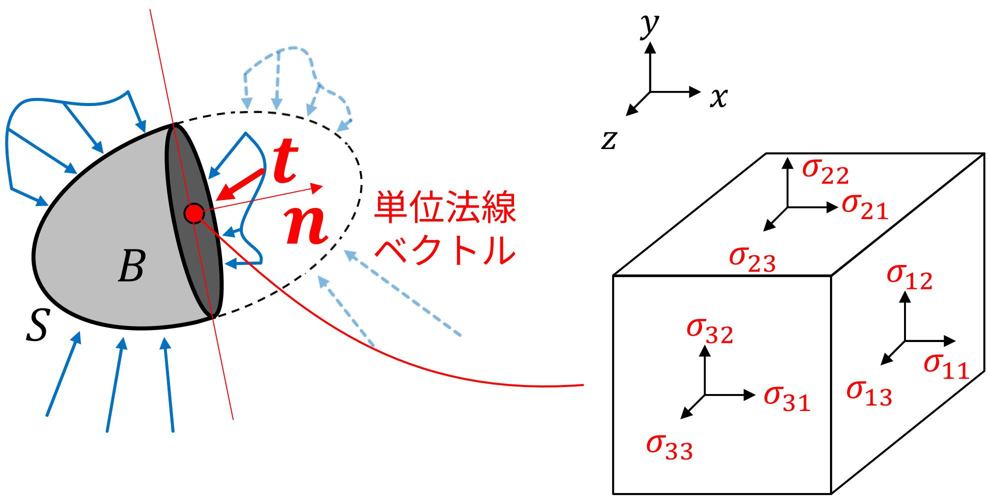
応力ベクトルは
切断面の方向に依存する



切断面は
単位法線ベクトルで定義する

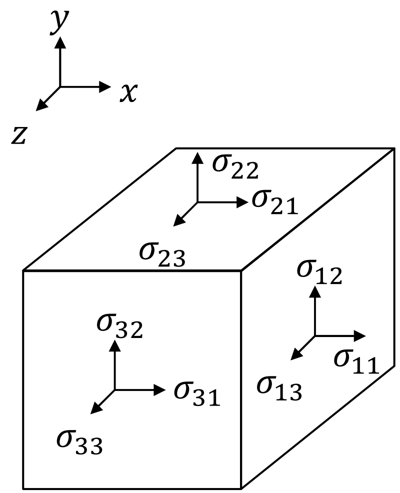


応力状態は、切断面の全方向に対応できる量



応力ベクトルは
応力状態と切断面から導く

$$\boldsymbol{t} = f(\sigma, \boldsymbol{n})$$

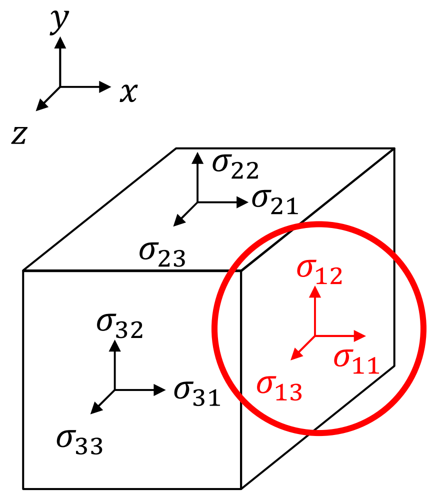


切断面がx面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix}$$

$$\boldsymbol{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\boxed{\boldsymbol{t} = f(\sigma, \boldsymbol{n})}$$

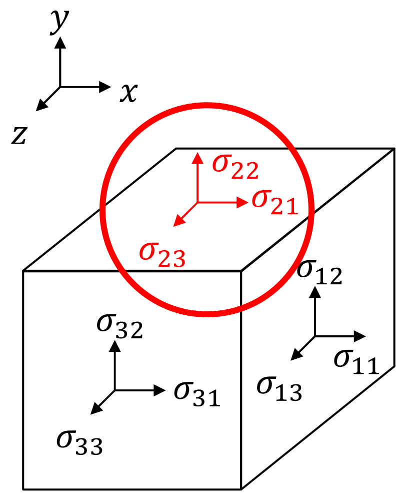


切断面がx面　切断面がy面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} \qquad \boldsymbol{t} = \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix}$$

$$\boldsymbol{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$\boxed{\boldsymbol{t} = f(\sigma, \boldsymbol{n})}$$

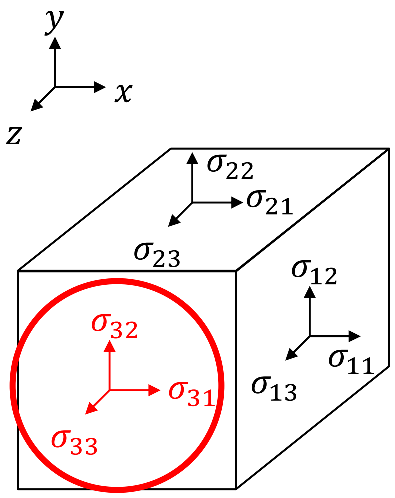


切断面がx面　切断面がy面　切断面がz面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} \qquad \boldsymbol{t} = \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} \qquad \boldsymbol{t} = \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix}$$

$$\boldsymbol{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$\boxed{\boldsymbol{t} = f(\sigma, \boldsymbol{n})}$$

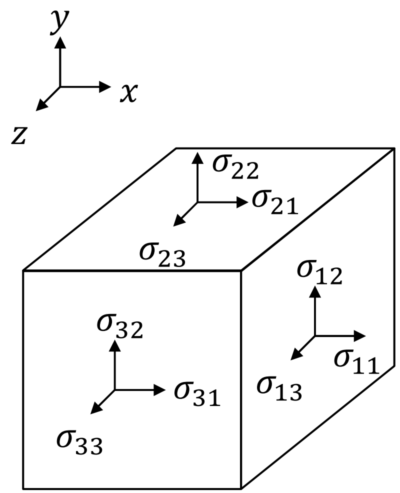


切断面がx面　切断面がy面　切断面がz面　任意の切断面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} \quad \boldsymbol{t} = \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} \quad \boldsymbol{t} = \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix} \quad \boldsymbol{t} = n_1 \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} + n_2 \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} + n_3 \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix}$$

$$\boldsymbol{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \quad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \quad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \quad \boldsymbol{n} = \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} \quad |\boldsymbol{n}| = \sqrt{n_1^2 + n_2^2 + n_3^2} = 1$$

$$\boxed{\boldsymbol{t} = f(\sigma, \boldsymbol{n})}$$



切断面がx面

$$\boldsymbol{t}=\begin{Bmatrix}\sigma_{11}\\ \sigma_{12}\\ \sigma_{13}\end{Bmatrix}$$

$$\boldsymbol{n}=\begin{Bmatrix}1\\ 0\\ 0\end{Bmatrix}$$

切断面がy面

$$\boldsymbol{t}=\begin{Bmatrix}\sigma_{21}\\ \sigma_{22}\\ \sigma_{23}\end{Bmatrix}$$

$$\boldsymbol{n}=\begin{Bmatrix}0\\ 1\\ 0\end{Bmatrix}$$

切断面がz面

$$\boldsymbol{t}=\begin{Bmatrix}\sigma_{31}\\ \sigma_{32}\\ \sigma_{33}\end{Bmatrix}$$

$$\boldsymbol{n}=\begin{Bmatrix}0\\ 0\\ 1\end{Bmatrix}$$

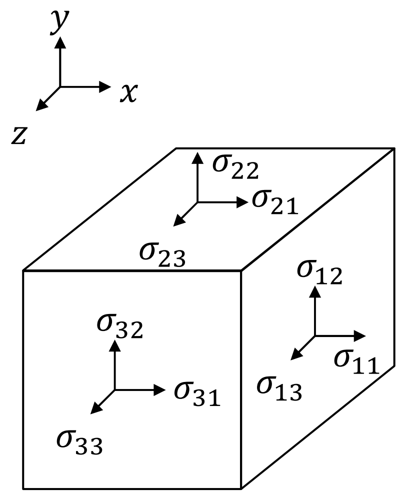
任意の切断面

$$\boldsymbol{t}=n_1\begin{Bmatrix}\sigma_{11}\\ \sigma_{12}\\ \sigma_{13}\end{Bmatrix}+n_2\begin{Bmatrix}\sigma_{21}\\ \sigma_{22}\\ \sigma_{23}\end{Bmatrix}+n_3\begin{Bmatrix}\sigma_{31}\\ \sigma_{32}\\ \sigma_{33}\end{Bmatrix}$$

$$\boldsymbol{n}=\begin{Bmatrix}n_1\\ n_2\\ n_3\end{Bmatrix}$$

$$|\boldsymbol{n}|=\sqrt{n_1^2+n_2^2+n_3^2}=1$$

$$\boxed{\boldsymbol{t}=f(\sigma,\boldsymbol{n})}$$

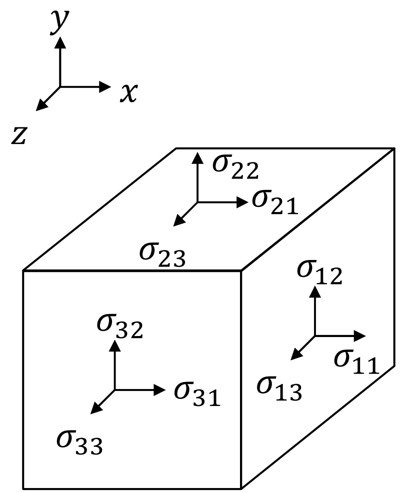


切断面がx面　切断面がy面　切断面がz面　任意の切断面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} \qquad \boldsymbol{t} = \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} \qquad \boldsymbol{t} = \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix} \qquad \boldsymbol{t} = n_1 \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} + n_2 \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} + n_3 \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix}$$

$$\boldsymbol{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix}$$

$$|\boldsymbol{n}| = \sqrt{n_1^2 + n_2^2 + n_3^2} = 1$$



$$\boxed{\boldsymbol{t} = f(\sigma, \boldsymbol{n})}$$

切断面がx面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

切断面がy面

$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

切断面がz面

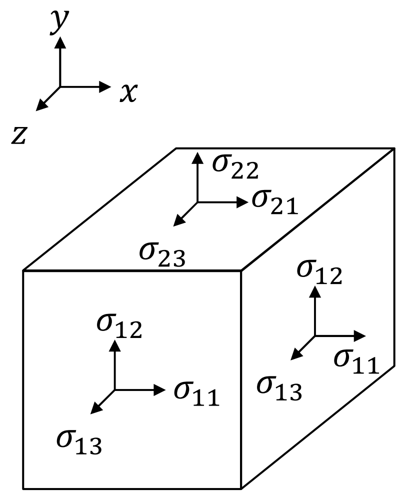
$$\boldsymbol{t} = \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

任意の切断面

$$\boxed{\boldsymbol{t} = n_1 \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} + n_2 \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} + n_3 \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix}}$$

$$\boldsymbol{n} = \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix}$$

$$\boxed{\begin{aligned} \boldsymbol{t} &= f(\sigma, \boldsymbol{n}) \\ &= \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}^{\mathrm{T}} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} \end{aligned}}$$



$$\boldsymbol{t} = n_1 \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{Bmatrix} + n_2 \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix} + n_3 \begin{Bmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix}$$

$$\Downarrow$$

$$\boldsymbol{t} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}^{\mathrm{T}} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix}$$

応力テンソルの
マトリクス形式

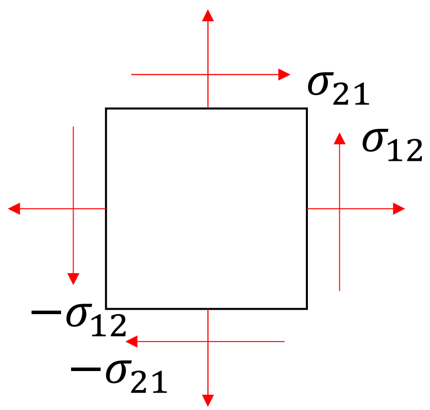
トラクションは

$$\textcolor{red}{\boldsymbol{t}} = [\sigma]^{\mathrm{T}}\{\boldsymbol{n}\}$$

Cauchy 応力テンソル
のマトリクス形式

切断面の
単位法線ベクトル

応力テンソルの
マトリクス形式は対称行列



$$\sigma_{ij} = \sigma_{ji}$$

応力テンソルの
転置を外しても同じ結果

$$\boldsymbol{t} = [\sigma]\{\boldsymbol{n}\}$$

$\boxed{\int_S \boldsymbol{t}\, da + \int_B \boldsymbol{b}\, dv = \boldsymbol{0}}$ に $\boxed{\boldsymbol{t} = [\sigma]^{\mathrm{T}}\{\boldsymbol{n}\}}$ を代入

$$\int_S [\sigma]^{\mathrm{T}} \boldsymbol{n}\, da + \int_B \boldsymbol{b}\, dv = \boldsymbol{0}$$

発散定理

$$\int_S [\quad] \cdot \boldsymbol{n}\, da = \int_B [\quad] \cdot \boldsymbol{\nabla}\, dv$$

$$\begin{Bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{Bmatrix}$$

発散定理

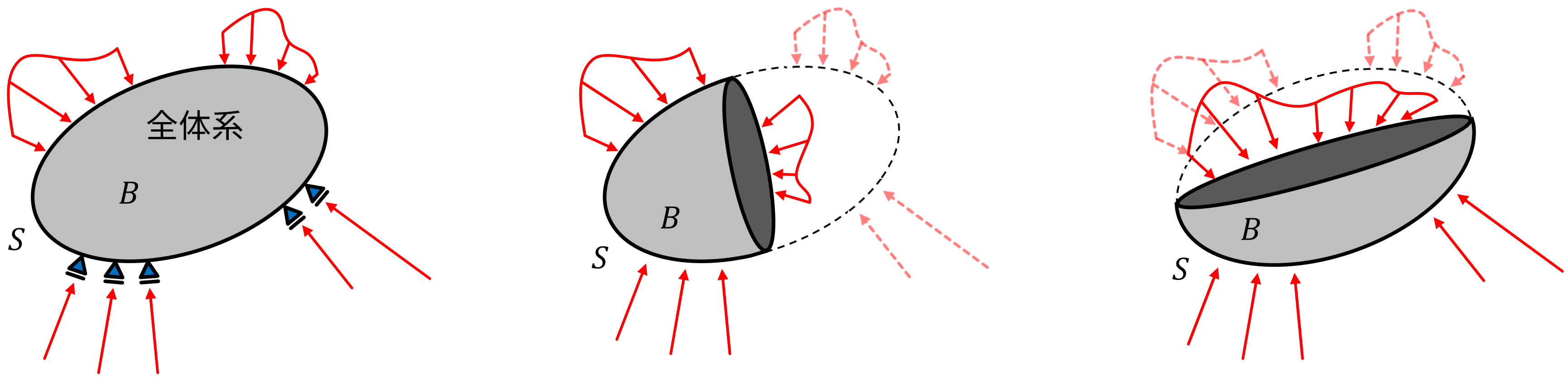
$$\int_S [\sigma]^{\mathrm{T}} \boldsymbol{n}\, da = \int_B \begin{Bmatrix} \dfrac{\partial \sigma_{11}}{\partial x_1} + \dfrac{\partial \sigma_{21}}{\partial x_2} + \dfrac{\partial \sigma_{31}}{\partial x_3} \\ \dfrac{\partial \sigma_{12}}{\partial x_1} + \dfrac{\partial \sigma_{22}}{\partial x_2} + \dfrac{\partial \sigma_{32}}{\partial x_3} \\ \dfrac{\partial \sigma_{13}}{\partial x_1} + \dfrac{\partial \sigma_{23}}{\partial x_2} + \dfrac{\partial \sigma_{33}}{\partial x_3} \end{Bmatrix} dv$$

$$= \int_B \left\{ \frac{\partial \sigma_{ji}}{\partial x_j} \right\} dv$$

j: dummy index

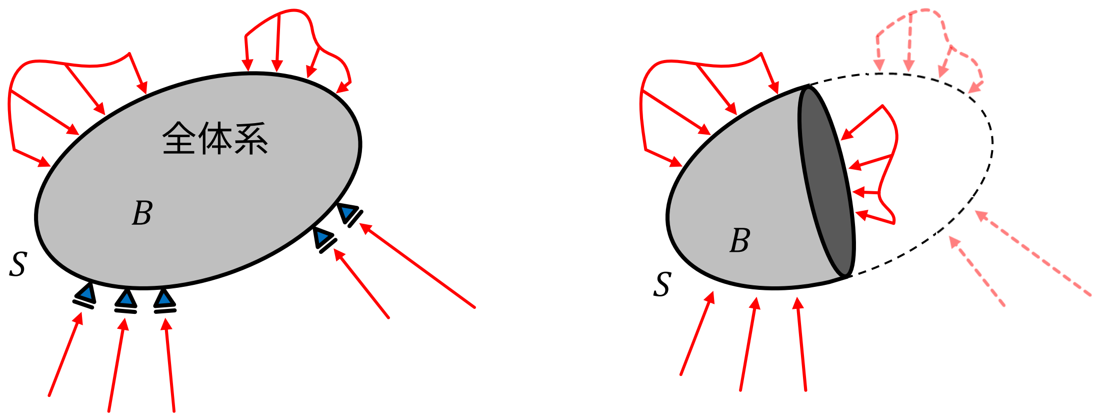
力のつりあい式

$$\int_B \left\{ \frac{\partial \sigma_{ji}}{\partial x_j} \right\} + \boldsymbol{b}\, dv = \boldsymbol{0}$$



力のつりあい式

$$\left\{\frac{\partial \sigma_{ji}}{\partial x_j}\right\} + \boldsymbol{b} = \boldsymbol{0}$$



力のつりあい式

$$\frac{\partial \sigma_{ji}}{\partial x_j} + b_i = 0 \quad \Longrightarrow \quad \boxed{\frac{\partial \sigma_{ji}}{\partial x_j} = 0}$$

$b_i = 0$

体積力は無視する
場合が多い

微分方程式を
有限要素法で
数値的に解く

対象は力のつりあい式

荷重条件
境界条件

$$\boxed{\frac{\partial \sigma_{ji}}{\partial x_j} = 0}$$

$$\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix}$$

変位

数値**解**を代入

荷重条件
境界条件

$$\boxed{\frac{\partial \sigma_{ji}}{\partial x_j} = r_i}$$

数値解

$$\begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

代入

残差式を得る

残差を最小化する数値解を求める。 ＝ **重み付き残差の全体平均をゼロとする数値解**

$$\int_{\underset{\text{全体系の領域}}{B}} \underbrace{\boldsymbol{\omega}}_{\text{重みベクトル}} \underset{\text{内積}}{\cdot} \underbrace{\boldsymbol{r}}_{\text{残差ベクトル}} \,\mathrm{d}v = 0$$

力のつりあい式を
重み付き残差式に代入

$$\int_B \left\{\omega_i\right\} \cdot \left\{\frac{\partial \sigma_{ji}}{\partial x_j}\right\} \mathrm{d}v = 0$$

さらに式を変形する

$$\int_B \omega_i \frac{\partial \sigma_{ji}}{\partial x_j} \mathrm{d}v$$

は

$$\int_B \frac{\partial \omega_i \sigma_{ji}}{\partial x_j} \mathrm{d}v = \int_B \omega_i \frac{\partial \sigma_{ji}}{\partial x_j} \mathrm{d}v + \int_B \frac{\partial \omega_i}{\partial x_j} \sigma_{ji} \mathrm{d}v$$

に含まれる

$$\int_B \omega_i \frac{\partial \sigma_{ji}}{\partial x_j} \mathrm{d}v = 0$$

は

発散定理より

$$\boxed{\int_S \omega_i \sigma_{ji} n_j \mathrm{d}a} = \int_B \omega_i \frac{\partial \sigma_{ji}}{\partial x_j} \mathrm{d}v + \int_B \frac{\partial \omega_i}{\partial x_j} \sigma_{ji} \mathrm{d}v$$

に含まれる

力のつりあい式は

$$\int_S \omega_i \sigma_{ji} n_j \mathrm{d}a = \int_B \frac{\partial \omega_i}{\partial x_j} \sigma_{ji} \mathrm{d}v$$

$$\int_S \{\boldsymbol{\omega}\} \cdot \boxed{\left(\left[\sigma \right]^{\mathrm{T}} \{\boldsymbol{n}\} \right)} \mathrm{d}a = \int_B \left[\frac{\partial \omega_i}{\partial x_j} \right] : \left[\sigma_{ji} \right] \mathrm{d}v$$

$$\| $$

$$\{\boldsymbol{t}\}$$

（テンソル内積）

各成分の積の総和

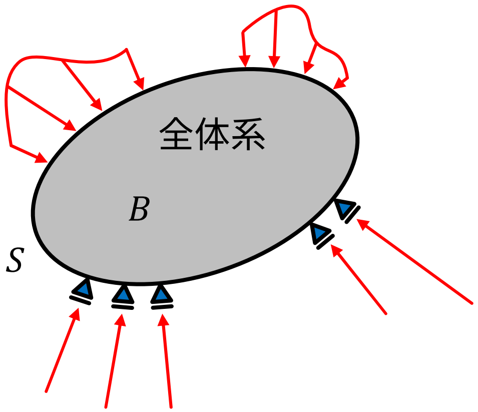
$$\frac{\partial \omega_1}{\partial x_1}\sigma_{11} + \frac{\partial \omega_1}{\partial x_2}\sigma_{21} + \frac{\partial \omega_1}{\partial x_3}\sigma_{31} + \frac{\partial \omega_2}{\partial x_1}\sigma_{12} + \cdots$$

重み付き残差式は全体系で定義される

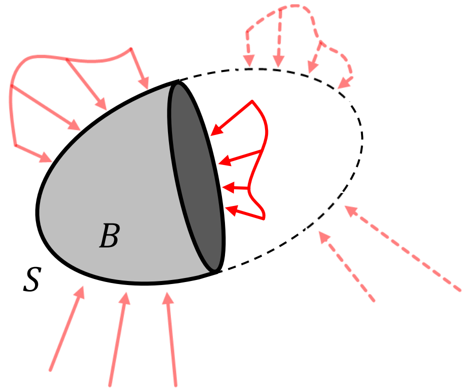
$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\} \mathrm{d}a = \int_B \left[\frac{\partial \omega_i}{\partial x_j}\right] : \left[\sigma_{ji}\right] \mathrm{d}v$$

全体系表面の
トラクション

全領域
の応力



左辺は外力　右辺は内力



全体系表面の
トラクション

↓

外力
- 集中荷重
- 分布荷重

変位と重みを近似解に置き換えて解く

$$\int_S \{\boldsymbol{\omega}\}\cdot\{\boldsymbol{t}\}\,\mathrm{d}a = \int_B \left[\frac{\partial\omega_i}{\partial x_j}\right] : [\sigma_{ji}]\,\mathrm{d}v$$

代入する前に
弱形式化すること

代入

近似解

$$\begin{array}{cccccc} & & & \begin{array}{ccc} n & n & n \end{array} & & \\ 3 & \{\boldsymbol{\omega}\} & = & \begin{bmatrix} \boldsymbol{N}^{\mathrm{T}} & & \\ & \boldsymbol{N}^{\mathrm{T}} & \\ & & \boldsymbol{N}^{\mathrm{T}} \end{bmatrix} & \begin{Bmatrix} \boldsymbol{\omega}_1 \\ \boldsymbol{\omega}_2 \\ \boldsymbol{\omega}_3 \end{Bmatrix} & 3n \end{array}$$

$$\begin{array}{cccccc} 3 & \{\boldsymbol{u}\} & = & \begin{bmatrix} \boldsymbol{N}^{\mathrm{T}} & & \\ & \boldsymbol{N}^{\mathrm{T}} & \\ & & \boldsymbol{N}^{\mathrm{T}} \end{bmatrix} & \begin{Bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \\ \boldsymbol{u}_3 \end{Bmatrix} & 3n \end{array}$$

$\boldsymbol{u}$ はσ_{ji}を変形
すると出てくる

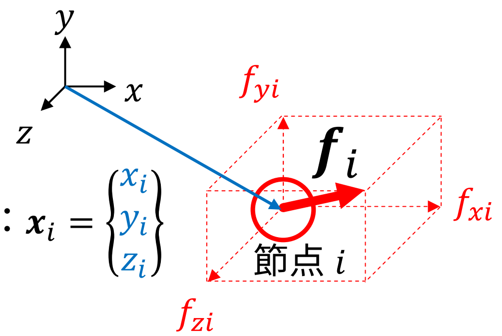
最終的に、有限要素式の左辺は
各自由度の荷重ベクトルとなる

$$\int_S \{\boldsymbol{\omega}\}\cdot\{\boldsymbol{t}\}\,\mathrm{d}a = \int_B \left[\frac{\partial\omega_i}{\partial x_j}\right]:\left[\sigma_{ji}\right]\mathrm{d}v$$

$$\boldsymbol{f}_x = \begin{Bmatrix}\vdots\\ f_{xi}\\ \vdots\end{Bmatrix},\quad \boldsymbol{f}_y = \begin{Bmatrix}\vdots\\ f_{yi}\\ \vdots\end{Bmatrix},\quad \boldsymbol{f}_z = \begin{Bmatrix}\vdots\\ f_{zi}\\ \vdots\end{Bmatrix}$$

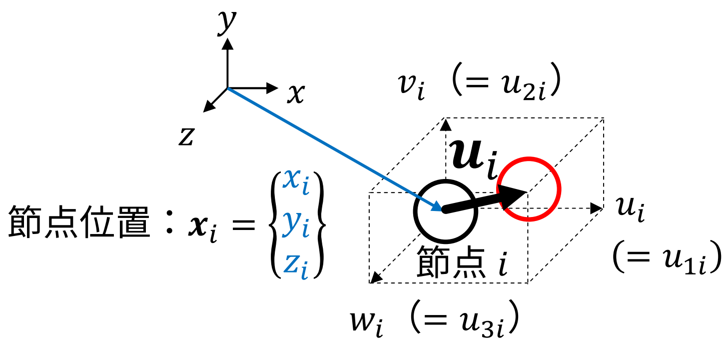
$$\begin{bmatrix}\{\boldsymbol{f}_x\}\\ \{\boldsymbol{f}_y\}\\ \{\boldsymbol{f}_z\}\end{bmatrix}$$

節点位置：$\boldsymbol{x}_i = \begin{Bmatrix}x_i\\ y_i\\ z_i\end{Bmatrix}$



一方で、有限要素式の右辺は剛性と変位ベクトルとなる

$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\}\, \mathrm{d}a = \int_B \left[\frac{\partial \omega_i}{\partial x_j}\right] : \left[\sigma_{ji}\right] \mathrm{d}v$$

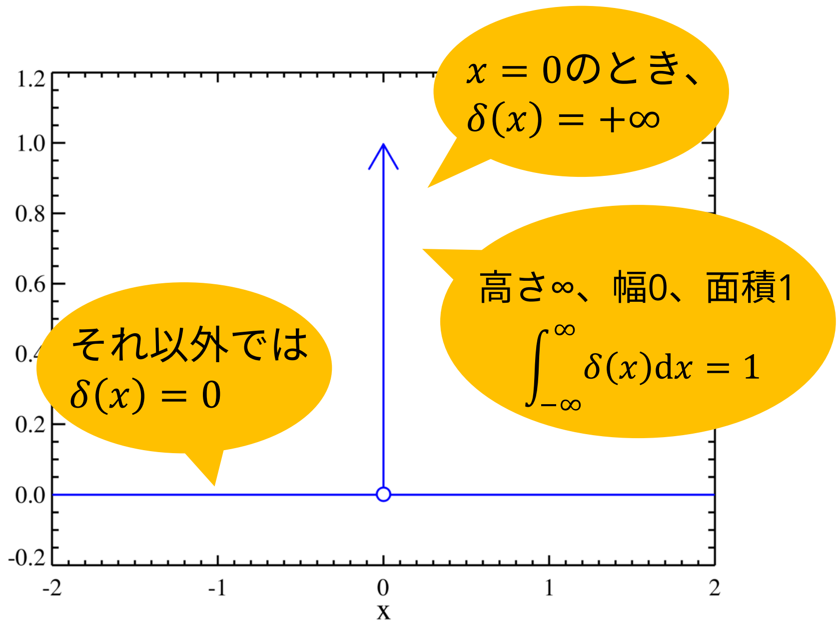


$$\Downarrow$$

$$\begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix} \begin{bmatrix} \{\boldsymbol{u}_1\} \\ \{\boldsymbol{u}_2\} \\ \{\boldsymbol{u}_3\} \end{bmatrix}$$

$$\boldsymbol{u}_1 = \begin{Bmatrix} \vdots \\ u_{1i} \\ \vdots \end{Bmatrix}, \quad \boldsymbol{u}_2 = \begin{Bmatrix} \vdots \\ u_{2i} \\ \vdots \end{Bmatrix}, \quad \boldsymbol{u}_3 = \begin{Bmatrix} \vdots \\ u_{3i} \\ \vdots \end{Bmatrix}$$

集中荷重はデルタ関数



デュラックのデルタ関数

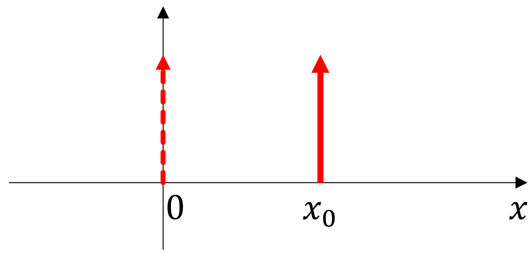
デルタ関数の**利**用方法

任意の関数

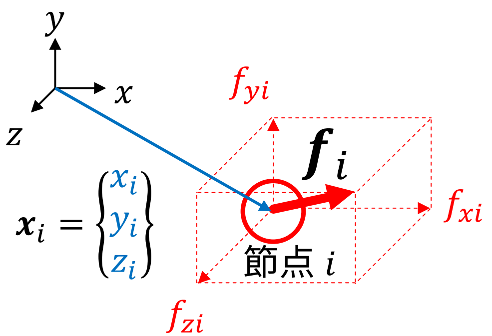
$$\int_{-\infty}^{\infty} \boxed{\delta(x - x_0)} F(x)\mathrm{d}x = F(x_0)$$

$x = x_0$における
関数の値を返す

$x = x_0$にあるインパルス



全体系表面のトラクションは全て節点上の集中荷重とする



節点位置：$\boldsymbol{x}_i = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}$

$$\{\boldsymbol{t}\} = \begin{Bmatrix} \cdots + \delta(\boldsymbol{x} - \boldsymbol{x}_i) f_{xi} + \cdots \\ \cdots + \delta(\boldsymbol{x} - \boldsymbol{x}_i) f_{yi} + \cdots \\ \cdots + \delta(\boldsymbol{x} - \boldsymbol{x}_i) f_{zi} + \cdots \end{Bmatrix}$$

トラクション

デルタ関数を
3次元空間に拡張

ほとんどの項が対応する点を指定しないとゼロになる
※　$\boldsymbol{x} = \boldsymbol{x}_i$で、$\{\boldsymbol{t}\} = \{\boldsymbol{f}_i\}$

内挿を用いた
重みの近似解

$$\omega_i = \boldsymbol{\omega}_i^T \boldsymbol{N}$$

$N_j(x, y, z)$は節点jの時だけ1
それ以外の節点では0となる関数

3×1　n×1

$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\}\,\mathrm{d}a = [\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T] \boxed{\int_S \begin{bmatrix} \{\boldsymbol{N}\} & & \\ & \{\boldsymbol{N}\} & \\ & & \{\boldsymbol{N}\} \end{bmatrix} \{\boldsymbol{t}\}\,\mathrm{d}a}$$

全体系表面の
トラクション

各自由度
における
重みの値

$\boldsymbol{t}$ は 節点位置で インパルス を持つδ関数 × 各方向における その節点位置での 集中荷重の値

3（次元数）

$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\} \, \mathrm{d}a = [\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T] \int_S \begin{bmatrix} \boxed{\{\boldsymbol{N}\}} & & \\ & \{\boldsymbol{N}\} & \\ & & \{\boldsymbol{N}\} \end{bmatrix} \boxed{\{\boldsymbol{t}\}} \, \mathrm{d}a$$

自由度数：3n

$$\begin{Bmatrix} \sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{xk} \\ \sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{yk} \\ \sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{zk} \end{Bmatrix}$$

jが1～nまで変化
しながら、縦に並ぶ

$$= [\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T] \begin{Bmatrix} \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{xk} \right) \mathrm{d}a \\ \vdots \\ \vdots \\ \vdots \end{Bmatrix}$$

自由度数：3n

3（次元数）

$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\} \mathrm{d}a = \underbrace{[\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T]}_{\text{自由度数：3n}} \boxed{\int_S \begin{bmatrix} \{\boldsymbol{N}\} & & \\ & \boxed{\{\boldsymbol{N}\}} & \\ & & \{\boldsymbol{N}\} \end{bmatrix} \boxed{\{\boldsymbol{t}\}} \mathrm{d}a}$$

jが1～nまで変化
しながら、縦に並ぶ

$$= [\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T] \left\{ \begin{array}{c} \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{xk} \right) \mathrm{d}a \\ \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{yk} \right) \mathrm{d}a \\ \vdots \\ \\ \vdots \end{array} \right\}$$

自由度数：3n

3（次元数）

$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\} \mathrm{d}a = \underbrace{[\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T]}_{\text{自由度数：3n}} \boxed{\int_S \begin{bmatrix} \{\boldsymbol{N}\} & & \\ & \{\boldsymbol{N}\} & \\ & & \{\boldsymbol{N}\} \end{bmatrix} \boxed{\{\boldsymbol{t}\}} \mathrm{d}a}$$

jが1〜nまで変化しながら、縦に並ぶ

$$= [\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T] \begin{Bmatrix} \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{xk} \right) \mathrm{d}a \\ \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{yk} \right) \mathrm{d}a \\ \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{zk} \right) \mathrm{d}a \\ \vdots \end{Bmatrix}$$

自由度数：3n

3（次元数）

$$\int_S \{\boldsymbol{\omega}\} \cdot \{\boldsymbol{t}\}\,\mathrm{d}a = \underbrace{[\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T]}_{\text{自由度数：3n}} \boxed{\int_S \begin{bmatrix} \{\boldsymbol{N}\} & & \\ & \{\boldsymbol{N}\} & \\ & & \{\boldsymbol{N}\} \end{bmatrix} \{\boldsymbol{t}\}\,\mathrm{d}a}$$

jが1～nまで変化
しながら、縦に並ぶ

$$= [\boldsymbol{\omega}_1^T \quad \boldsymbol{\omega}_2^T \quad \boldsymbol{\omega}_3^T] \begin{Bmatrix} \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{xk} \right) \mathrm{d}a \\ \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{yk} \right) \mathrm{d}a \\ \vdots \\ \int_S N_j(x,y,z) \cdot \left(\sum_k^n \delta(\boldsymbol{x} - \boldsymbol{x}_k) \cdot f_{zk} \right) \mathrm{d}a \\ \vdots \end{Bmatrix}$$

自由度数：3n

節点(x_j, y_j, z_j)の時だけ１
それ以外の節点ではゼロ

節点 j と節点 k が一致
しないとゼロ

$$\int_S N_j(x,y,z)\cdot\left(\sum_k^n \delta(\boldsymbol{x}-\boldsymbol{x}_k)\cdot f_{xk}\right)\mathrm{d}a$$

$$=\int_S\left(\sum_k^n \delta(\boldsymbol{x}-\boldsymbol{x}_k)\cdot N_j(x,y,z)f_{xk}\right)\mathrm{d}a$$

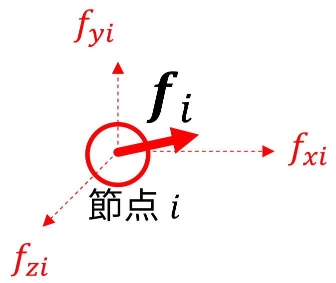
$$=\left(\sum_k^n \boxed{N_j(x_k,y_k,z_k)}f_{xk}\right)$$

$$=f_{xj}$$

節点(x_j, y_j, z_j)の時だけ１
それ以外の節点ではゼロ

残るのは各節点の荷重ベクトル

$$\int_S \{\boldsymbol{w}\} \cdot \{\boldsymbol{t}\} \, \mathrm{d}a = [\boldsymbol{W}_1^T \quad \boldsymbol{W}_2^T \quad \boldsymbol{W}_3^T] \begin{bmatrix} \{\boldsymbol{f}_{\boldsymbol{x}}\} \\ \{\boldsymbol{f}_{\boldsymbol{y}}\} \\ \{\boldsymbol{f}_{\boldsymbol{z}}\} \end{bmatrix}$$



後は右辺をうまく処理すれば解ける

$$
\begin{bmatrix} W_1^T & W_2^T & W_3^T \end{bmatrix}
\begin{bmatrix} \{\boldsymbol{f}_x\} \\ \{\boldsymbol{f}_y\} \\ \{\boldsymbol{f}_z\} \end{bmatrix}
= \boxed{\int_B \left[\frac{\partial \omega_i}{\partial x_j}\right] : [\sigma_{ji}] \mathrm{d}v}
$$

$$
= \begin{bmatrix} W_1^T & W_2^T & W_3^T \end{bmatrix}
\begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix}
\begin{bmatrix} \{\boldsymbol{U}\} \\ \{\boldsymbol{V}\} \\ \{\boldsymbol{W}\} \end{bmatrix}
$$

⬇

全体系表面の
トラクション
(つまり外力)

$$
\begin{bmatrix} \{\boldsymbol{f}_x\} \\ \{\boldsymbol{f}_y\} \\ \{\boldsymbol{f}_z\} \end{bmatrix}
= \begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix}
\begin{bmatrix} \{\boldsymbol{U}\} \\ \{\boldsymbol{V}\} \\ \{\boldsymbol{W}\} \end{bmatrix}
$$

重み関数

i は x、y、z方向

次元数＝3（方向）

節点の数：n

$$\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial N_n}{\partial x} & \frac{\partial N_n}{\partial y} & \frac{\partial N_n}{\partial z} \end{bmatrix} \begin{Bmatrix} \sigma_{12} \\ \sigma_{22} \\ \sigma_{32} \end{Bmatrix}$$

$$\int_B \left[\frac{\partial w_i}{\partial x_j}\right] : [\sigma_{ji}] \mathrm{d}v = \int_B \left[\frac{\partial (\boldsymbol{W}_i \cdot \boldsymbol{N})}{\partial x_j}\right] : [\sigma_{ji}] \mathrm{d}v$$

$$= \boldsymbol{W}_1^{\mathrm{T}} \int_B \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] \{\sigma_{j1}\} \mathrm{d}v + \boldsymbol{W}_2^{\mathrm{T}} \int_B \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] \{\sigma_{j2}\} \mathrm{d}v + \boldsymbol{W}_3^{\mathrm{T}} \int_B \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] \{\sigma_{j3}\} \mathrm{d}v$$

テンソル内積

$[A_{ij}]:[B_{ij}]$

$= A_{11}B_{11} + A_{12}B_{12} + A_{13}B_{13}$

$+ A_{21}B_{21} + A_{22}B_{22} + A_{23}B_{23}$

$+ A_{31}B_{31} + A_{32}B_{32} + A_{33}B_{33}$

$$= [\boldsymbol{W}_1^{\mathrm{T}} \quad \boldsymbol{W}_2^{\mathrm{T}} \quad \boldsymbol{W}_3^{\mathrm{T}}] \int_B \begin{bmatrix} \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] & & \\ & \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] & \\ & & \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{21} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma_{22} \\ \sigma_{32} \\ \sigma_{13} \\ \sigma_{23} \\ \sigma_{33} \end{Bmatrix} \mathrm{d}v$$

あとは順序を
整理するだけ

$$\int_B \left[\frac{\partial w_i}{\partial x_j}\right] : [\sigma_{ji}] \mathrm{d}v = [\boldsymbol{W}_1^{\mathrm{T}} \quad \boldsymbol{W}_2^{\mathrm{T}} \quad \boldsymbol{W}_3^{\mathrm{T}}] \int_B \begin{bmatrix} \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] & & \\ & \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] & \\ & & \left[\frac{\partial \boldsymbol{N}}{\partial x_j}\right] \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{21} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma_{22} \\ \sigma_{32} \\ \sigma_{13} \\ \sigma_{23} \\ \sigma_{33} \end{Bmatrix} \mathrm{d}v$$

$$= [\boldsymbol{W}_1^{\mathrm{T}} \quad \boldsymbol{W}_2^{\mathrm{T}} \quad \boldsymbol{W}_3^{\mathrm{T}}] \int_B \boxed{\begin{bmatrix} \frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x} \end{bmatrix}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} \mathrm{d}v$$

Bマトリクス

$$
\begin{bmatrix} \boldsymbol{W}_1^T & \boldsymbol{W}_2^T & \boldsymbol{W}_3^T \end{bmatrix}
\begin{bmatrix} \{\boldsymbol{f}_x\} \\ \{\boldsymbol{f}_y\} \\ \{\boldsymbol{f}_z\} \end{bmatrix}
= \begin{bmatrix} \boldsymbol{W}_1^{\mathrm{T}} & \boldsymbol{W}_2^{\mathrm{T}} & \boldsymbol{W}_3^{\mathrm{T}} \end{bmatrix}
\int_B
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} \mathrm{d}v
$$

$$
\begin{bmatrix} \{\boldsymbol{f}_x\} \\ \{\boldsymbol{f}_y\} \\ \{\boldsymbol{f}_z\} \end{bmatrix}
= \int_B
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} \mathrm{d}v
$$

右辺は、各積分点における数値積分

$$
\begin{bmatrix} \{\boldsymbol{f}_x\} \\ \{\boldsymbol{f}_y\} \\ \{\boldsymbol{f}_z\} \end{bmatrix} = \int_B \begin{bmatrix} \frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} \mathrm{d}v
$$

各点の応力を、各点の変位に変換したい

応力－歪関係にフックの法則を仮定

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \underbrace{\begin{bmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{bmatrix}}_{\mathbf{D}} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}$$

各点の応力を、各点の歪に変換

歪－変位関係に微小変形理論を適用

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & & \\ & \frac{\partial}{\partial y} & \\ & & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & \\ & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

変位に近似解を代入

$$
\begin{aligned}
u &= \boldsymbol{u}\cdot\boldsymbol{N} & v &= \boldsymbol{v}\cdot\boldsymbol{N} & w &= \boldsymbol{w}\cdot\boldsymbol{N}\\
&= \boldsymbol{N}^{\mathrm{T}}\boldsymbol{u} & &= \boldsymbol{N}^{\mathrm{T}}\boldsymbol{v} & &= \boldsymbol{N}^{\mathrm{T}}\boldsymbol{w}
\end{aligned}
$$

$$
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}
=
\begin{bmatrix}
\frac{\partial}{\partial x} & & \\
& \frac{\partial}{\partial y} & \\
& & \frac{\partial}{\partial z} \\
\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & \\
& \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\
\frac{\partial}{\partial z} & & \frac{\partial}{\partial x}
\end{bmatrix}
\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}
=
\underbrace{\begin{bmatrix}
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial x} & & \\
& \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial y} & \\
& & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial z} \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial x} & \\
& \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial y} \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial z} & & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial x}
\end{bmatrix}}_{\mathbf{B}^{\mathrm{T}}}
\begin{bmatrix} \{\boldsymbol{u}\} \\ \{\boldsymbol{v}\} \\ \{\boldsymbol{w}\} \end{bmatrix}
$$

$$\{\boldsymbol{f}\} = \int_B [\mathbf{B}]\{\boldsymbol{\sigma}\}\,\mathrm{d}v$$

$$= \int_B [\mathbf{B}][\mathbf{D}]\{\boldsymbol{\varepsilon}\}\,\mathrm{d}v$$

$$= \int_B [\mathbf{B}][\mathbf{D}][\mathbf{B}]^{\mathrm{T}}\{\boldsymbol{u}\}\mathrm{d}v$$

$$= \int_B [\mathbf{B}][\mathbf{D}][\mathbf{B}]^{\mathrm{T}}\mathrm{d}v\,\{\boldsymbol{u}\}$$

$$\begin{bmatrix}\{\boldsymbol{f}_x\}\\ \{\boldsymbol{f}_y\}\\ \{\boldsymbol{f}_z\}\end{bmatrix} = \int_B \begin{bmatrix}\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z}\\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}\end{bmatrix}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{Bmatrix}\mathrm{d}v$$

$$= \int_B \begin{bmatrix}\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z}\\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}\end{bmatrix}\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\end{Bmatrix}\mathrm{d}v$$

$$= \left(\int_B \begin{bmatrix}\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z}\\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}\end{bmatrix}\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu\end{bmatrix}\begin{bmatrix}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial x} & & \\ & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial y} & \\ & & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial z}\\ \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial x} & \\ & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial y}\\ \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial z} & & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial x}\end{bmatrix}\mathrm{d}v\right)\begin{bmatrix}\{\boldsymbol{u}\}\\ \{\boldsymbol{v}\}\\ \{\boldsymbol{w}\}\end{bmatrix}$$

剛性マトリクス[K]を計算できたら数値解が求まる

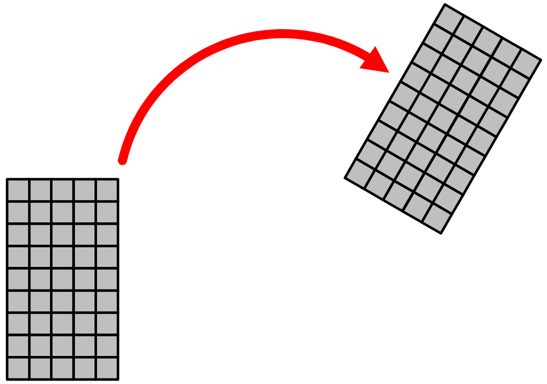
$$\{\boldsymbol{f}\} = \boxed{\int_B [\mathbf{B}][\mathbf{D}][\mathbf{B}]^{\mathrm{T}} \mathrm{d}v} \{\boldsymbol{u}\} \Rightarrow \boxed{\{\boldsymbol{f}\} = [\mathbf{K}]\{\boldsymbol{u}\}}$$

境界条件（荷重と変位の一部が与えられる）
代入後、連立方程式を解く（**逆行列問題**）

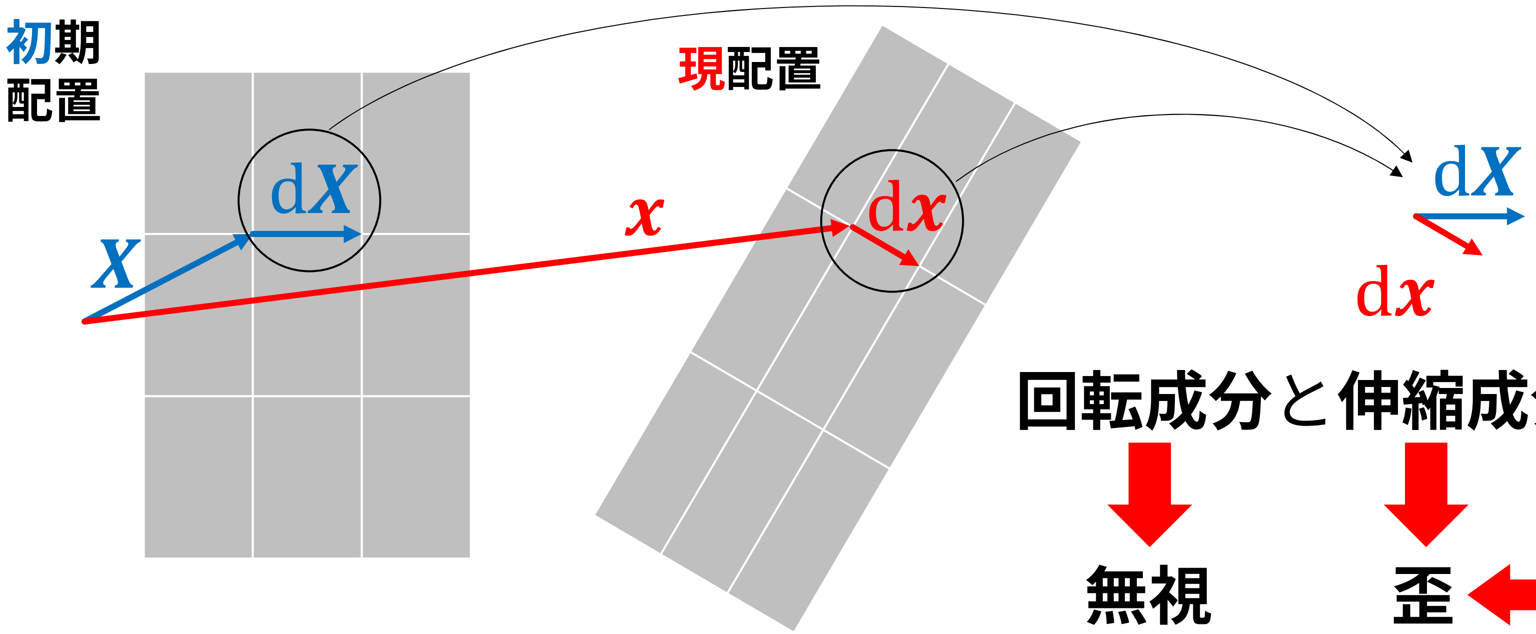
ただし、微小変形が前提

微小変形歪は剛体回転でゼロにならない

$$
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}
=
\begin{bmatrix}
\frac{\partial}{\partial x} & & \\
 & \frac{\partial}{\partial y} & \\
 & & \frac{\partial}{\partial z} \\
\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & \\
 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\
\frac{\partial}{\partial z} & & \frac{\partial}{\partial x}
\end{bmatrix}
\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}
$$



剛体移動・剛体回転は
ゼロになるように歪を再定義したい



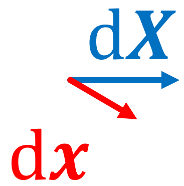
回転成分と伸縮成分に分解

無視　　歪 ⟷ 応力

（大変形対応）

変形勾配テンソル

$$
\begin{Bmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \\ \mathrm{d}x_3 \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x_1}{\partial X_1} & \dfrac{\partial x_1}{\partial X_2} & \dfrac{\partial x_1}{\partial X_3} \\ \dfrac{\partial x_2}{\partial X_1} & \dfrac{\partial x_2}{\partial X_2} & \dfrac{\partial x_2}{\partial X_3} \\ \dfrac{\partial x_3}{\partial X_1} & \dfrac{\partial x_3}{\partial X_2} & \dfrac{\partial x_3}{\partial X_3} \end{bmatrix} \begin{Bmatrix} \mathrm{d}X_1 \\ \mathrm{d}X_2 \\ \mathrm{d}X_3 \end{Bmatrix}
$$



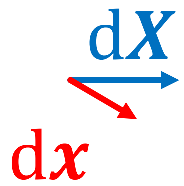
全微分の変数変換

$$dx = \frac{\partial x}{\partial X} dX + \frac{\partial x}{\partial Y} dY + \frac{\partial x}{\partial Z} dZ$$

$$\begin{Bmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \\ \mathrm{d}x_3 \end{Bmatrix} = \underbrace{\boxed{\begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix}}}_{\mathbf{F}} \begin{Bmatrix} \mathrm{d}X_1 \\ \mathrm{d}X_2 \\ \mathrm{d}X_3 \end{Bmatrix}$$

変形勾配テンソル

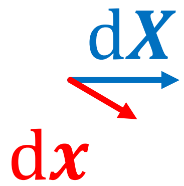
$$\{\mathrm{d}x_i\} = \left[\frac{\partial x_i}{\partial X_j}\right]\{\mathrm{d}X_j\}$$



j: dummy index

変形勾配テンソル

$$\mathrm{d}\boldsymbol{x} = \mathbf{F}\mathrm{d}\boldsymbol{X}$$



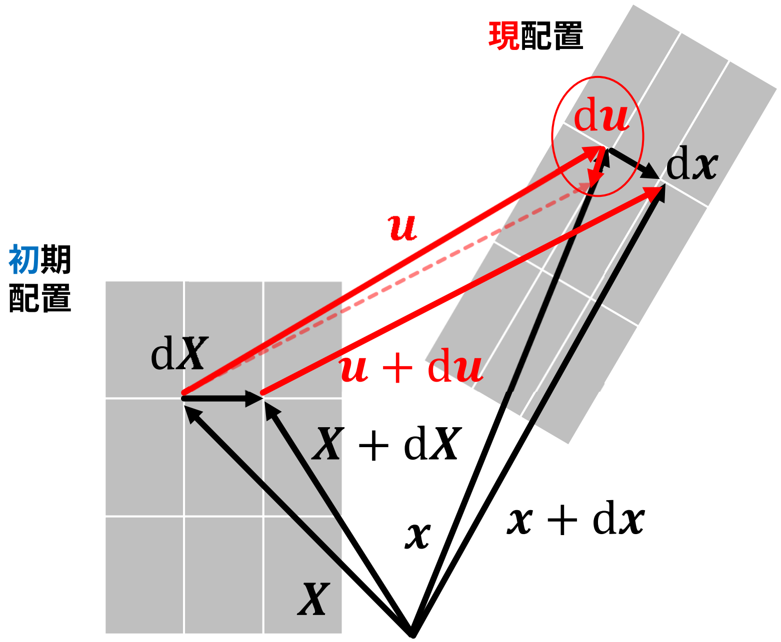
変位ベクトルと変形勾配テンソル

$$\boldsymbol{u} = \boldsymbol{x} - \boldsymbol{X}$$

$$\boldsymbol{u} + \mathrm{d}\boldsymbol{u} = (\boldsymbol{x} + \mathrm{d}\boldsymbol{x}) - (\boldsymbol{X} + \mathrm{d}\boldsymbol{X})$$
$$= \boldsymbol{x} - \boldsymbol{X} + \mathrm{d}\boldsymbol{x} - \mathrm{d}\boldsymbol{X}$$
$$= \boldsymbol{u} + \mathrm{d}\boldsymbol{x} - \mathrm{d}\boldsymbol{X}$$

$$\mathrm{d}\boldsymbol{u} = \mathrm{d}\boldsymbol{x} - \mathrm{d}\boldsymbol{X}$$
$$= \mathbf{F}\mathrm{d}\boldsymbol{X} - \mathrm{d}\boldsymbol{X} = (\mathbf{F} - \mathbf{I})\mathrm{d}\boldsymbol{X}$$

$\mathrm{d}\boldsymbol{x} = \mathbf{F}\mathrm{d}\boldsymbol{X}$



$$\boxed{\mathrm{d}\boldsymbol{u} = (\mathbf{F} - \mathbf{I})\mathrm{d}\boldsymbol{X}} \Longleftrightarrow \boxed{\mathrm{d}\boldsymbol{u} = \left[\frac{\partial u_i}{\partial X_j}\right]\mathrm{d}\boldsymbol{X}}$$

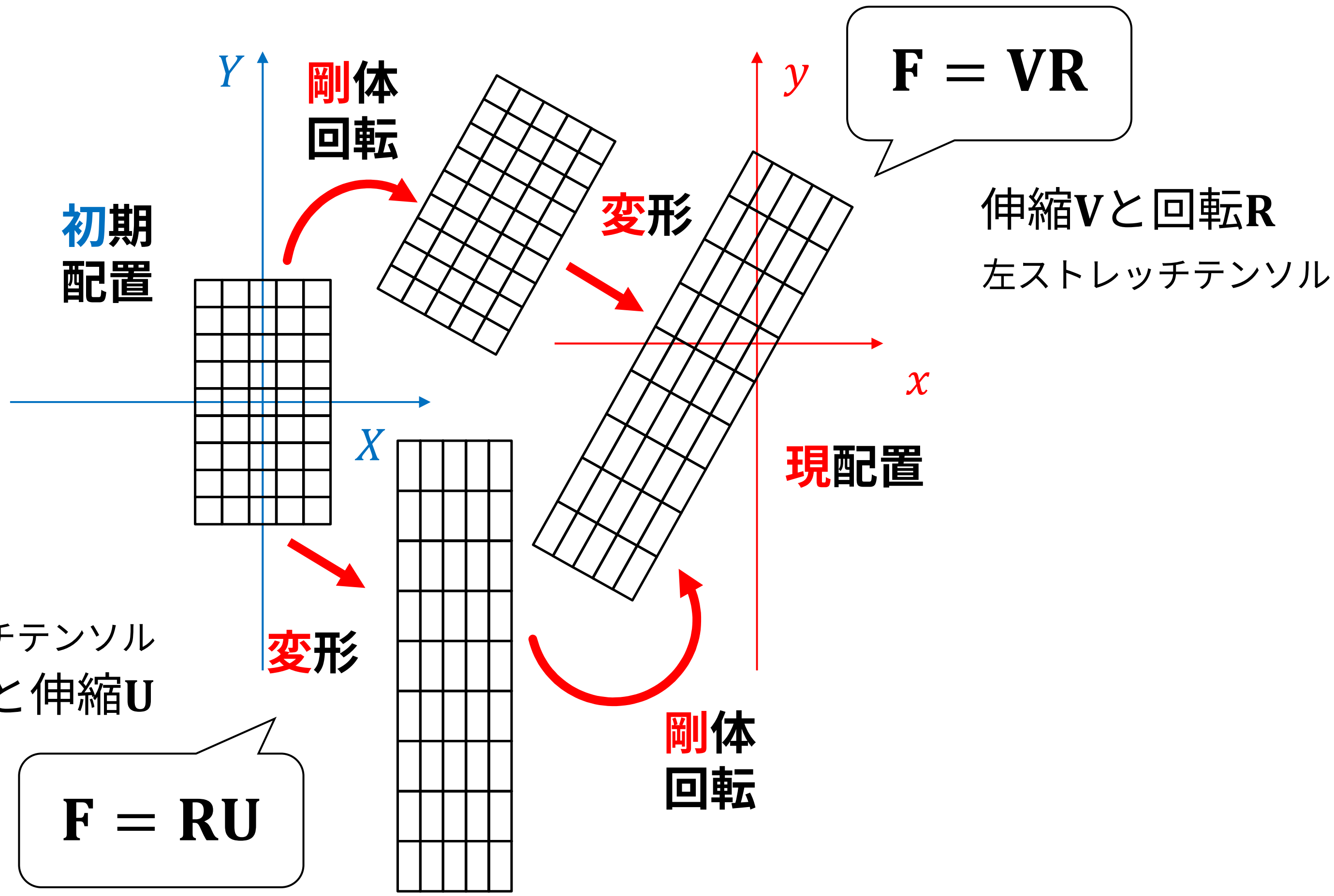
$$\mathbf{F} - \mathbf{I} = \left[\frac{\partial u_i}{\partial X_j}\right]$$

$$\boxed{\mathbf{F} = \left[\frac{\partial u_i}{\partial X_j}\right] + \mathbf{I}}$$

$$F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

$\mathrm{d}\boldsymbol{x} = \mathbf{F}\mathrm{d}\boldsymbol{X}$

変形勾配**F**は
2通りに分解できる



応力を生じるのは伸縮テンソル

$$\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}$$

回転テンソルを消したい

対称行列

$\mathbf{U} = \mathbf{U}^{\mathrm{T}}$

$\mathbf{V} = \mathbf{V}^{\mathrm{T}}$

直交行列

$\mathbf{R}^{-1} = \mathbf{R}^{\mathrm{T}}$

$$\mathbf{F} = \mathbf{R}\mathbf{U}$$

$$\mathbf{F}^{\mathrm{T}}\mathbf{F} = \mathbf{U}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{R}\mathbf{U} = \mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{U}^{2}$$

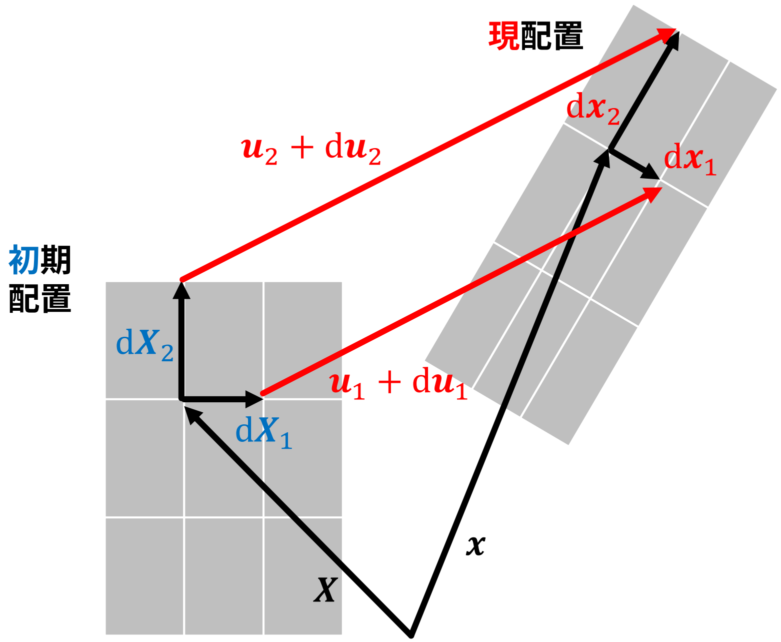
$$\mathbf{F} = \mathbf{V}\mathbf{R}$$

$$\mathbf{F}\mathbf{F}^{\mathrm{T}} = \mathbf{V}\mathbf{R}\mathbf{R}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} = \mathbf{V}\mathbf{V}^{\mathrm{T}} = \mathbf{V}^{2}$$

Green-Lagrange歪

$$\mathrm{d}\boldsymbol{x}_1 \cdot \mathrm{d}\boldsymbol{x}_2 = (\mathbf{F}\mathrm{d}\boldsymbol{X}_1) \cdot (\mathbf{F}\mathrm{d}\boldsymbol{X}_2)$$
$$= \mathrm{d}\boldsymbol{X}_1^{\mathrm{T}}\mathbf{F}^{\mathrm{T}}\mathbf{F}\mathrm{d}\boldsymbol{X}_2$$
$$= \mathrm{d}\boldsymbol{X}_1^{\mathrm{T}}\mathbf{U}^2\mathrm{d}\boldsymbol{X}_2$$

$$\mathbf{U}^2 = \mathbf{F}^{\mathrm{T}}\mathbf{F} = \left(\left[\frac{\partial u_j}{\partial X_i}\right] + \mathbf{I}\right)\left(\left[\frac{\partial u_i}{\partial X_j}\right] + \mathbf{I}\right)$$
$$= \Bigg(\underbrace{\boxed{\left[\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right] + \left[\frac{\partial u_j}{\partial X_i}\right] + \left[\frac{\partial u_i}{\partial X_j}\right]}}_{2E_{ij}} + \underbrace{\mathbf{I}}_{\text{元の長さ}}\Bigg)$$



全微分の変数変換

$$dx = \frac{\partial x}{\partial X}dX + \frac{\partial x}{\partial Y}dY + \frac{\partial x}{\partial Z}dZ$$

$$\begin{Bmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \\ \mathrm{d}x_3 \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix}}_{\mathbf{F}} \begin{Bmatrix} \mathrm{d}X_1 \\ \mathrm{d}X_2 \\ \mathrm{d}X_3 \end{Bmatrix}$$

偏微分の変数変換

$$\frac{\partial \omega}{\partial x} = \frac{\partial \omega}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial \omega}{\partial Y}\frac{\partial Y}{\partial x} + \frac{\partial \omega}{\partial Z}\frac{\partial Z}{\partial x}$$

$$\begin{Bmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3 \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \frac{\partial X_2}{\partial x_1} & \frac{\partial X_3}{\partial x_1} \\ \frac{\partial X_1}{\partial x_2} & \frac{\partial X_2}{\partial x_2} & \frac{\partial X_3}{\partial x_2} \\ \frac{\partial X_1}{\partial x_3} & \frac{\partial X_2}{\partial x_3} & \frac{\partial X_3}{\partial x_3} \end{bmatrix}}_{\mathbf{F}^{-\mathrm{T}}} \begin{Bmatrix} \partial/\partial X_1 \\ \partial/\partial X_2 \\ \partial/\partial X_3 \end{Bmatrix}$$

変形勾配テンソル

$$\begin{Bmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \\ \mathrm{d}x_3 \end{Bmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix} \begin{Bmatrix} \mathrm{d}X_1 \\ \mathrm{d}X_2 \\ \mathrm{d}X_3 \end{Bmatrix} \qquad \begin{Bmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3 \end{Bmatrix} = \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \frac{\partial X_2}{\partial x_1} & \frac{\partial X_3}{\partial x_1} \\ \frac{\partial X_1}{\partial x_2} & \frac{\partial X_2}{\partial x_2} & \frac{\partial X_3}{\partial x_2} \\ \frac{\partial X_1}{\partial x_3} & \frac{\partial X_2}{\partial x_3} & \frac{\partial X_3}{\partial x_3} \end{bmatrix} \begin{Bmatrix} \partial/\partial X_1 \\ \partial/\partial X_2 \\ \partial/\partial X_3 \end{Bmatrix}$$

$$\begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix} \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \frac{\partial X_2}{\partial x_1} & \frac{\partial X_3}{\partial x_1} \\ \frac{\partial X_1}{\partial x_2} & \frac{\partial X_2}{\partial x_2} & \frac{\partial X_3}{\partial x_2} \\ \frac{\partial X_1}{\partial x_3} & \frac{\partial X_2}{\partial x_3} & \frac{\partial X_3}{\partial x_3} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix} \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \frac{\partial X_1}{\partial x_2} & \frac{\partial X_1}{\partial x_3} \\ \frac{\partial X_2}{\partial x_1} & \frac{\partial X_2}{\partial x_2} & \frac{\partial X_2}{\partial x_3} \\ \frac{\partial X_3}{\partial x_1} & \frac{\partial X_3}{\partial x_2} & \frac{\partial X_3}{\partial x_3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix} \begin{bmatrix} \frac{\partial X_1}{\partial x_1} & \frac{\partial X_1}{\partial x_2} & \frac{\partial X_1}{\partial x_3} \\ \frac{\partial X_2}{\partial x_1} & \frac{\partial X_2}{\partial x_2} & \frac{\partial X_2}{\partial x_3} \\ \frac{\partial X_3}{\partial x_1} & \frac{\partial X_3}{\partial x_2} & \frac{\partial X_3}{\partial x_3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

対角成分

$$\frac{\partial x_1}{\partial X_1}\frac{\partial X_1}{\partial x_1} + \frac{\partial x_1}{\partial X_2}\frac{\partial X_2}{\partial x_1} + \frac{\partial x_1}{\partial X_3}\frac{\partial X_3}{\partial x_1}$$

$$= \frac{x_1}{x_1} = 1$$

非対角成分

$$\frac{\partial x_1}{\partial X_1}\frac{\partial X_1}{\partial x_2} + \frac{\partial x_1}{\partial X_2}\frac{\partial X_2}{\partial x_2} + \frac{\partial x_1}{\partial X_3}\frac{\partial X_3}{\partial x_2}$$

$$= \frac{x_1}{x_2} = 0$$

Nansonの公式

２つのベクトルの外積は座標変換後どうなるか？

$\boldsymbol{x}$と$\boldsymbol{y}$を、$\boldsymbol{a}$と$\boldsymbol{b}$にFで変換

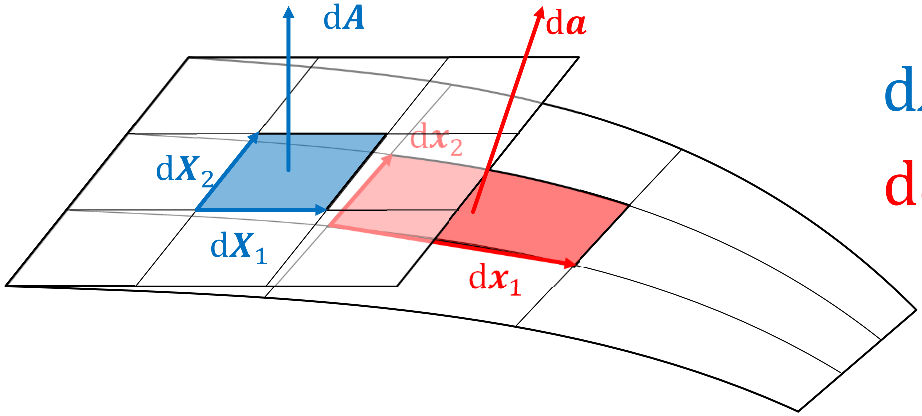
$$\boxed{\boldsymbol{a}\times\boldsymbol{b} = (\mathbf{F}\boldsymbol{x})\times(\mathbf{F}\boldsymbol{y})}$$

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \times \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} = \begin{Bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{Bmatrix} = \begin{Bmatrix} F_{2j}x_j F_{3l}y_l - F_{3j}x_j F_{2l}y_l \\ F_{3j}x_j F_{1l}y_l - F_{1j}x_j F_{3l}y_l \\ F_{1j}x_j F_{2l}y_l - F_{2j}x_j F_{1l}y_l \end{Bmatrix} = \begin{Bmatrix} F_{2i}F_{3j}\left(x_i y_j - x_j y_i\right) \\ F_{3i}F_{1j}\left(x_i y_j - x_j y_i\right) \\ F_{1i}F_{2j}\left(x_i y_j - x_j y_i\right) \end{Bmatrix}$$

$$\boxed{a_i = F_{ij}x_j \qquad b_k = F_{kl}y_l}$$

$$= \begin{bmatrix} F_{22}F_{33} - F_{23}F_{32} & F_{21}F_{33} - F_{23}F_{31} & F_{22}F_{33} - F_{23}F_{32} \\ F_{32}F_{13} - F_{33}F_{12} & F_{31}F_{13} - F_{33}F_{31} & F_{31}F_{32} - F_{32}F_{31} \\ F_{12}F_{23} - F_{13}F_{22} & F_{11}F_{23} - F_{13}F_{21} & F_{11}F_{22} - F_{12}F_{21} \end{bmatrix} \begin{Bmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{Bmatrix}$$

$$\boxed{= J\mathbf{F}^{-\mathrm{T}}(\boldsymbol{x}\times\boldsymbol{y})}$$

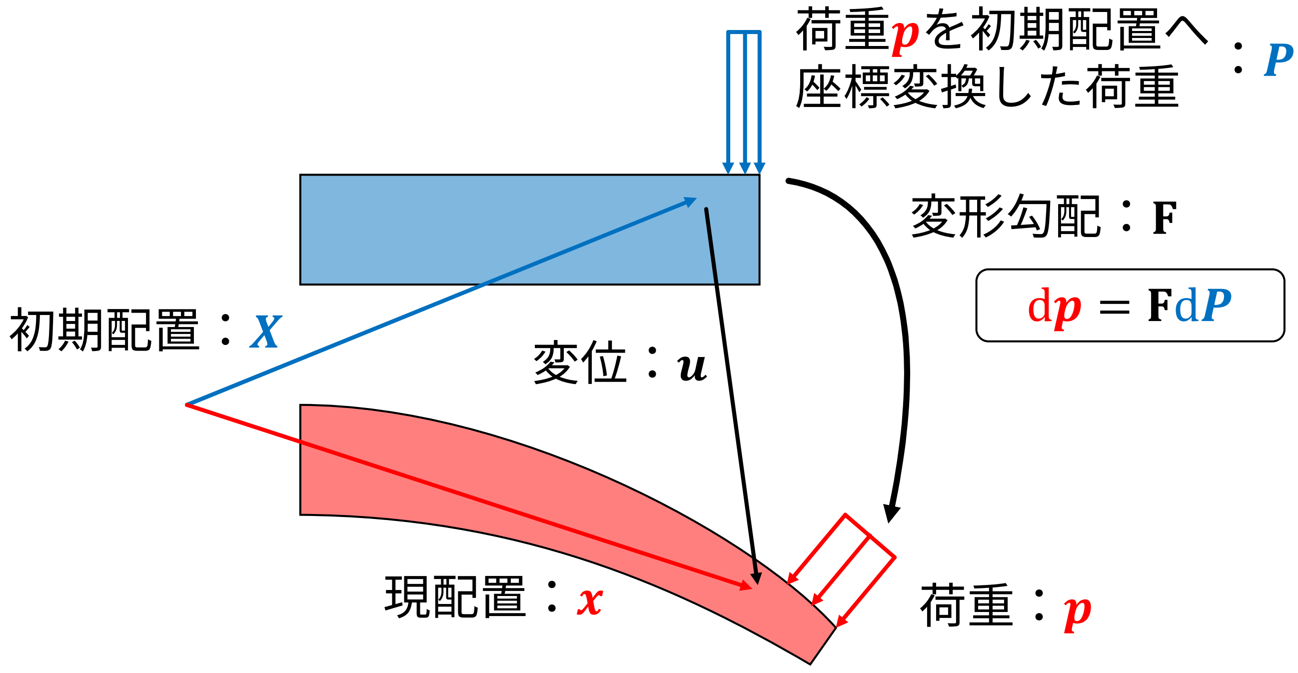


Nansonの公式

$$\mathrm{d}\boldsymbol{A} = \boldsymbol{N}\mathrm{d}A = \mathrm{d}\boldsymbol{X}_1 \times \mathrm{d}\boldsymbol{X}_2$$

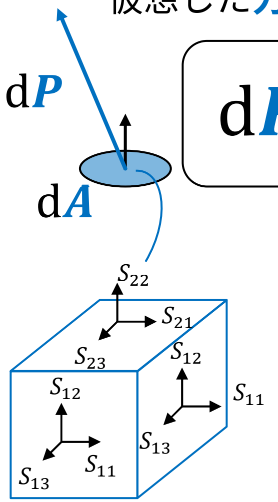
$$\begin{aligned}\mathrm{d}\boldsymbol{a} = \boldsymbol{n}\mathrm{d}a &= \mathrm{d}\boldsymbol{x}_1 \times \mathrm{d}\boldsymbol{x}_2 \\ &= (\mathbf{F}\mathrm{d}\boldsymbol{X}_1) \times (\mathbf{F}\mathrm{d}\boldsymbol{X}_2) \\ &= J\mathbf{F}^{-\mathrm{T}}(\mathrm{d}\boldsymbol{X}_1 \times \mathrm{d}\boldsymbol{X}_2) \\ &= J\mathbf{F}^{-\mathrm{T}}\mathrm{d}\boldsymbol{A}\end{aligned}$$

$$\boxed{\boldsymbol{n}\mathrm{d}a = J\mathbf{F}^{-\mathrm{T}}\boldsymbol{N}\mathrm{d}A}$$


荷重$\boldsymbol{p}$を初期配置へ
座標変換した荷重
：$\boldsymbol{P}$
変形勾配：$\mathbf{F}$
$\mathrm{d}\boldsymbol{p} = \mathbf{F}\mathrm{d}\boldsymbol{P}$
初期配置：$\boldsymbol{X}$
変位：$\boldsymbol{u}$
現配置：$\boldsymbol{x}$
荷重：$\boldsymbol{p}$

初期配置に
仮想した力

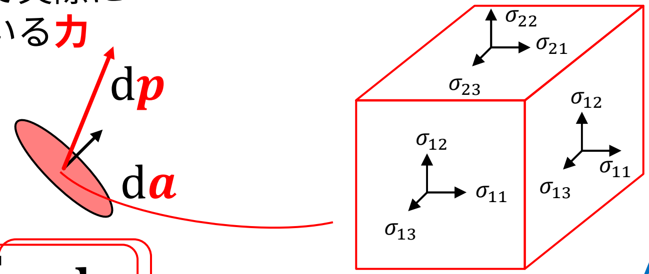
現配置で実際に
働いている力



$$\mathrm{d}\boldsymbol{P} = \mathbf{F}^{-1}\mathrm{d}\boldsymbol{p}$$
$$= \mathbf{F}^{-1}[\boldsymbol{\sigma}]^{\mathrm{T}}\boldsymbol{n}\mathrm{d}a$$

Nansonの公式

$$= \mathbf{F}^{-1}[\boldsymbol{\sigma}]^{\mathrm{T}}J\mathbf{F}^{-\mathrm{T}}\boldsymbol{N}\mathrm{d}A$$
$$= \mathbf{F}^{-1}[\boldsymbol{\pi}]^{\mathrm{T}}\mathbf{F}^{-\mathrm{T}}\boldsymbol{N}\mathrm{d}A$$
$$= \mathbf{F}^{-1}[\boldsymbol{\Pi}]^{\mathrm{T}}\boldsymbol{N}\mathrm{d}A$$
$$= [\mathbf{S}]^{\mathrm{T}}\boldsymbol{N}\mathrm{d}A$$



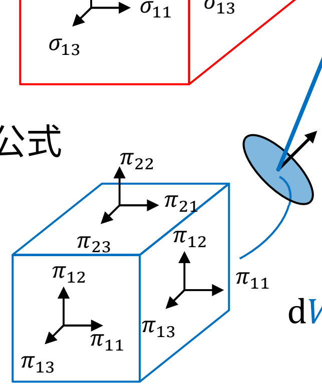
Cauchy応力 $[\boldsymbol{\sigma}]$

$$\mathrm{d}v = (\mathrm{d}\boldsymbol{x}_1 \times \mathrm{d}\boldsymbol{x}_2)\cdot \mathrm{d}\boldsymbol{x}_3$$

大きさだけ揃える

Kirchhoff応力

$$[\boldsymbol{\pi}] = J[\boldsymbol{\sigma}]$$



$$\begin{aligned}\mathrm{d}V &= (\mathrm{d}\boldsymbol{X}_1 \times \mathrm{d}\boldsymbol{X}_2)\cdot \mathrm{d}\boldsymbol{X}_3\\ &= J\mathbf{F}^{-\mathrm{T}}(\mathrm{d}\boldsymbol{x}_1 \times \mathrm{d}\boldsymbol{x}_2)\cdot \mathbf{F}\mathrm{d}\boldsymbol{x}_3\\ &= J(\mathrm{d}\boldsymbol{x}_1 \times \mathrm{d}\boldsymbol{x}_2)^{\mathrm{T}}\mathbf{F}^{-1}\mathbf{F}\mathrm{d}\boldsymbol{x}_3\\ &= J(\mathrm{d}\boldsymbol{x}_1 \times \mathrm{d}\boldsymbol{x}_2)^{\mathrm{T}}\mathrm{d}\boldsymbol{x}_3 = J\mathrm{d}v\end{aligned}$$

(公称応力)　　変形を元に戻す

第1Piola-Kirchhoff応力 $[\boldsymbol{\Pi}] = \mathbf{F}^{-1}[\boldsymbol{\pi}]$

(疑似応力)　　対称性を与える

第2Piola-Kirchhoff応力 $[\mathbf{S}] = [\boldsymbol{\Pi}]\mathbf{F}^{-\mathrm{T}}$

第2Piola-Kirchhoff応力

$$[\mathbf{S}] = J\mathbf{F}^{-1}[\boldsymbol{\sigma}]\mathbf{F}^{-\mathrm{T}}$$

$$[\boldsymbol{\sigma}] = \frac{1}{J}\mathbf{F}[\mathbf{S}]\mathbf{F}^{\mathrm{T}}$$

$$\sigma_{11} = F_{1i}S_{ij}F_{1j} = F_{11}S_{11}F_{11} + F_{11}S_{12}F_{12} + F_{11}S_{13}F_{13} + F_{12}S_{21}F_{11} + F_{12}S_{22}F_{12} + F_{12}S_{23}F_{13} + F_{13}S_{31}F_{11} + F_{13}S_{32}F_{12} + F_{13}S_{33}F_{13}$$

$$\sigma_{22} = F_{2i}S_{ij}F_{2j} = F_{21}S_{11}F_{21} + F_{21}S_{12}F_{22} + F_{21}S_{13}F_{23} + F_{22}S_{21}F_{21} + F_{22}S_{22}F_{22} + F_{22}S_{23}F_{23} + F_{23}S_{31}F_{21} + F_{23}S_{32}F_{22} + F_{23}S_{33}F_{23}$$

$$\sigma_{33} = F_{3i}S_{ij}F_{3j} = F_{31}S_{11}F_{31} + F_{31}S_{12}F_{32} + F_{31}S_{13}F_{33} + F_{32}S_{21}F_{31} + F_{32}S_{22}F_{32} + F_{32}S_{23}F_{33} + F_{33}S_{31}F_{31} + F_{33}S_{32}F_{32} + F_{33}S_{33}F_{33}$$

$$\sigma_{12} = F_{1i}S_{ij}F_{2j} = F_{11}S_{11}F_{21} + F_{11}S_{12}F_{22} + F_{11}S_{13}F_{23} + F_{12}S_{21}F_{21} + F_{12}S_{22}F_{22} + F_{12}S_{23}F_{23} + F_{13}S_{31}F_{21} + F_{13}S_{32}F_{22} + F_{13}S_{33}F_{23}$$

$$\sigma_{23} = F_{2i}S_{ij}F_{3j} = F_{21}S_{11}F_{31} + F_{21}S_{12}F_{32} + F_{21}S_{13}F_{33} + F_{22}S_{21}F_{31} + F_{22}S_{22}F_{32} + F_{22}S_{23}F_{33} + F_{23}S_{31}F_{31} + F_{23}S_{32}F_{32} + F_{23}S_{33}F_{33}$$

$$\sigma_{31} = F_{3i}S_{ij}F_{1j} = F_{31}S_{11}F_{11} + F_{31}S_{12}F_{12} + F_{31}S_{13}F_{13} + F_{32}S_{21}F_{11} + F_{32}S_{22}F_{12} + F_{32}S_{23}F_{13} + F_{33}S_{31}F_{11} + F_{33}S_{32}F_{12} + F_{33}S_{33}F_{13}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\ F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\ F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\ F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22} + F_{12}F_{21} & F_{12}F_{23} + F_{13}F_{22} & F_{11}F_{23} + F_{13}F_{21} \\ F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32} + F_{22}F_{31} & F_{22}F_{33} + F_{23}F_{32} & F_{21}F_{33} + F_{23}F_{31} \\ F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12} + F_{32}F_{11} & F_{32}F_{13} + F_{33}F_{12} & F_{31}F_{13} + F_{33}F_{11} \end{bmatrix} \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{12} \\ S_{23} \\ S_{31} \end{Bmatrix}$$

有限要素法の
力のつりあい式の右辺

$$\int_B \left[\frac{\partial w_i}{\partial x_j}\right] : [\sigma_{ji}]\mathrm{d}v = [\boldsymbol{W}_1^{\mathrm{T}} \quad \boldsymbol{W}_2^{\mathrm{T}} \quad \boldsymbol{W}_3^{\mathrm{T}}] \int_B \begin{bmatrix} \frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} \mathrm{d}v$$

代入

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\ F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\ F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\ F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\ F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\ F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11} \end{bmatrix} \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{12} \\ S_{23} \\ S_{31} \end{Bmatrix}$$

現配置のBマトリクス　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$
\boxed{F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}}
$$

現配置のBマトリクス　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$
F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}
$$

=

$\frac{\partial \boldsymbol{N}}{\partial x}F_{11}F_{11} + \frac{\partial \boldsymbol{N}}{\partial y}F_{11}F_{21} + \frac{\partial \boldsymbol{N}}{\partial z}F_{31}F_{11}$					

現配置のBマトリクス

真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
 & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
 & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$
\boxed{F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}}
$$

=

$\left(\frac{\partial \boldsymbol{N}}{\partial x}F_{11} + \frac{\partial \boldsymbol{N}}{\partial y}F_{21} + \frac{\partial \boldsymbol{N}}{\partial z}F_{31}\right)F_{11}$					

現配置のBマトリクス　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\left(\frac{\partial \mathbf{N}}{\partial x}\frac{\partial x}{\partial X} + \frac{\partial \mathbf{N}}{\partial y}\frac{\partial y}{\partial X} + \frac{\partial \mathbf{N}}{\partial z}\frac{\partial z}{\partial X}\right)\left(\frac{\partial u}{\partial X} + 1\right)$					

現配置のBマトリクス　　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$						

現配置のBマトリクス　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \mathbf{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Y}$				

現配置のBマトリクス　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22} + F_{12}F_{21} & F_{12}F_{23} + F_{13}F_{22} & F_{11}F_{23} + F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32} + F_{22}F_{31} & F_{22}F_{33} + F_{23}F_{32} & F_{21}F_{33} + F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12} + F_{32}F_{11} & F_{32}F_{13} + F_{33}F_{12} & F_{31}F_{13} + F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

$=$

$\frac{\partial \mathbf{N}}{\partial X}\left(\frac{\partial u}{\partial X} + 1\right)$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial u}{\partial Z}$			

現配置のBマトリクス　　　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
 & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
 & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$					

現配置のBマトリクス　　　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$				

現配置のBマトリクス　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$
F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}
$$

=

$\frac{\partial \mathbf{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial u}{\partial Z}$			
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial v}{\partial Z}$			

現配置のBマトリクス　　　　　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial x}2F_{11}F_{12}+\frac{\partial \boldsymbol{N}}{\partial y}(F_{11}F_{22}+F_{12}F_{21})+\frac{\partial \boldsymbol{N}}{\partial z}(F_{31}F_{12}+F_{32}F_{11})$		
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス　　　　真応力から疑似応力への変換マトリクス

$$\begin{bmatrix} \boxed{\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z}} \\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x} \end{bmatrix} \begin{bmatrix} F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\ F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\ F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\ F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\ F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\ F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11} \end{bmatrix}$$

$$\boxed{F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial x}(F_{11}F_{12}+F_{11}F_{12})+\frac{\partial \boldsymbol{N}}{\partial y}(F_{11}F_{22}+F_{12}F_{21})+\frac{\partial \boldsymbol{N}}{\partial z}(F_{31}F_{12}+F_{32}F_{11})$		
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス　　　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$
\boxed{F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}}
$$

=

$\frac{\partial \mathbf{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\left(\frac{\partial \mathbf{N}}{\partial x}F_{12}+\frac{\partial \mathbf{N}}{\partial y}F_{22}+\frac{\partial \mathbf{N}}{\partial z}F_{32}\right)F_{11}+\left(\frac{\partial \mathbf{N}}{\partial x}F_{11}+\frac{\partial \mathbf{N}}{\partial y}F_{21}+\frac{\partial \mathbf{N}}{\partial z}F_{31}\right)F_{12}$		
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス

真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$F_{ij} = \dfrac{\partial x_i}{\partial X_j}$ または $F_{ij} = \dfrac{\partial u_i}{\partial X_j} + \delta_{ij}$

=

$\frac{\partial \mathbf{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\left(\frac{\partial \mathbf{N}}{\partial x}\frac{\partial x}{\partial Y}+\frac{\partial \mathbf{N}}{\partial y}\frac{\partial y}{\partial Y}+\frac{\partial \mathbf{N}}{\partial z}\frac{\partial z}{\partial Y}\right)\left(\frac{\partial u}{\partial X}+1\right)+\left(\frac{\partial \mathbf{N}}{\partial x}\frac{\partial x}{\partial X}+\frac{\partial \mathbf{N}}{\partial y}\frac{\partial y}{\partial X}+\frac{\partial \mathbf{N}}{\partial z}\frac{\partial z}{\partial X}\right)\frac{\partial u}{\partial Y}$		
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス　　真応力から疑似応力への変換マトリクス

$$\begin{bmatrix} \frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\ & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\ & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x} \end{bmatrix} \begin{bmatrix} F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\ F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\ F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\ F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\ F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\ F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11} \end{bmatrix}$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Y}$		
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス

真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \mathbf{N}}{\partial x} & & & \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial z} \\
& \frac{\partial \mathbf{N}}{\partial y} & & \frac{\partial \mathbf{N}}{\partial x} & \frac{\partial \mathbf{N}}{\partial z} & \\
& & \frac{\partial \mathbf{N}}{\partial z} & & \frac{\partial \mathbf{N}}{\partial y} & \frac{\partial \mathbf{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

F_{12}でくくる場合と
F_{13}でくくる場合がある

=

$\frac{\partial \mathbf{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \mathbf{N}}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \mathbf{N}}{\partial X}\frac{\partial u}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial u}{\partial Y}+\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial u}{\partial Z}$	
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \mathbf{N}}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial \mathbf{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \mathbf{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \mathbf{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial N}{\partial x} & & & \frac{\partial N}{\partial y} & & \frac{\partial N}{\partial z} \\
& \frac{\partial N}{\partial y} & & \frac{\partial N}{\partial x} & \frac{\partial N}{\partial z} & \\
& & \frac{\partial N}{\partial z} & & \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij}=\frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij}=\frac{\partial u_i}{\partial X_j}+\delta_{ij}$$

=

$\frac{\partial N}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial N}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial N}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial N}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial N}{\partial X}\frac{\partial u}{\partial Y}$	$\frac{\partial N}{\partial Z}\frac{\partial u}{\partial Y}+\frac{\partial N}{\partial Y}\frac{\partial u}{\partial Z}$	$\frac{\partial N}{\partial Z}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial N}{\partial X}\frac{\partial u}{\partial Z}$
$\frac{\partial N}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial N}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial N}{\partial Z}\frac{\partial v}{\partial Z}$			
$\frac{\partial N}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial N}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial N}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

現配置のBマトリクス　　　　真応力から疑似応力への変換マトリクス

$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
 & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
 & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Z}$
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$		
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

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$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
 & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
 & & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & \boxed{2F_{12}F_{13}} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & \boxed{2F_{22}F_{23}} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & \boxed{2F_{32}F_{33}} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & \boxed{F_{12}F_{23}+F_{13}F_{22}} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & \boxed{F_{22}F_{33}+F_{23}F_{32}} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & \boxed{F_{32}F_{13}+F_{33}F_{12}} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$
\boxed{F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}}
$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Z}$
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial Z}$	
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

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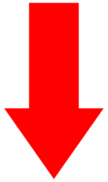
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$



$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Z}$
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial X}$
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$			

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$$
\begin{bmatrix}
\frac{\partial \boldsymbol{N}}{\partial x} & & & \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial z} \\
& \frac{\partial \boldsymbol{N}}{\partial y} & & \frac{\partial \boldsymbol{N}}{\partial x} & \frac{\partial \boldsymbol{N}}{\partial z} & \\
& & \frac{\partial \boldsymbol{N}}{\partial z} & & \frac{\partial \boldsymbol{N}}{\partial y} & \frac{\partial \boldsymbol{N}}{\partial x}
\end{bmatrix}
\begin{bmatrix}
F_{11}F_{11} & F_{12}F_{12} & F_{13}F_{13} & 2F_{11}F_{12} & 2F_{12}F_{13} & 2F_{11}F_{13} \\
F_{21}F_{21} & F_{22}F_{22} & F_{23}F_{23} & 2F_{21}F_{22} & 2F_{22}F_{23} & 2F_{21}F_{23} \\
F_{31}F_{31} & F_{32}F_{32} & F_{33}F_{33} & 2F_{31}F_{32} & 2F_{32}F_{33} & 2F_{31}F_{33} \\
F_{11}F_{21} & F_{12}F_{22} & F_{13}F_{23} & F_{11}F_{22}+F_{12}F_{21} & F_{12}F_{23}+F_{13}F_{22} & F_{11}F_{23}+F_{13}F_{21} \\
F_{21}F_{31} & F_{22}F_{32} & F_{23}F_{33} & F_{21}F_{32}+F_{22}F_{31} & F_{22}F_{33}+F_{23}F_{32} & F_{21}F_{33}+F_{23}F_{31} \\
F_{31}F_{11} & F_{32}F_{12} & F_{33}F_{13} & F_{31}F_{12}+F_{32}F_{11} & F_{32}F_{13}+F_{33}F_{12} & F_{31}F_{13}+F_{33}F_{11}
\end{bmatrix}
$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad \text{または} \quad F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$$

=

$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial u}{\partial X}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial u}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial u}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial u}{\partial X}+1\right)+\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial u}{\partial Z}$
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial v}{\partial Y}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial v}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial v}{\partial Y}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial Z}$	$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial v}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial v}{\partial X}$
$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial Z}\left(\frac{\partial w}{\partial Z}+1\right)$	$\frac{\partial \boldsymbol{N}}{\partial X}\frac{\partial w}{\partial Y}+\frac{\partial \boldsymbol{N}}{\partial Y}\frac{\partial w}{\partial X}$	$\frac{\partial \boldsymbol{N}}{\partial Y}\left(\frac{\partial w}{\partial Z}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial w}{\partial Y}$	$\frac{\partial \boldsymbol{N}}{\partial X}\left(\frac{\partial w}{\partial Z}+1\right)+\frac{\partial \boldsymbol{N}}{\partial Z}\frac{\partial w}{\partial X}$

つまり

$$\begin{Bmatrix}\boldsymbol{F}_X\\ \boldsymbol{F}_Y\\ \boldsymbol{F}_Z\end{Bmatrix}=\int_B\begin{bmatrix}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}\\ \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}\\ \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y}+\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z}+\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X}+\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z}\end{bmatrix}\begin{Bmatrix}s_{11}\\ s_{22}\\ s_{33}\\ s_{12}\\ s_{23}\\ s_{31}\end{Bmatrix}\mathrm{d}V$$

まとめて

$$\boxed{\begin{Bmatrix}\boldsymbol{F}_X\\ \boldsymbol{F}_Y\\ \boldsymbol{F}_Z\end{Bmatrix}=\int_B[\mathbf{B}_{\mathrm{L}}+\mathbf{B}_{\mathrm{N}}]\{S_{ij}\}\mathrm{d}V}$$

と書ける

解くには？

荷重
（ベクトル）

応力
（テンソル）

歪
（テンソル）

変位
（ベクトル）

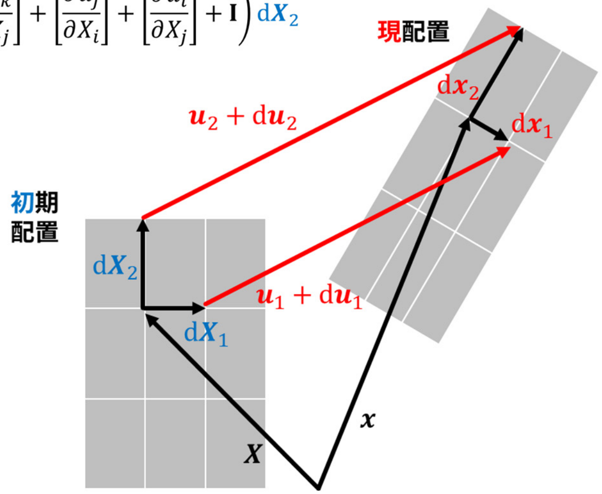
今ココ

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B [\mathbf{B}]\{S_{ij}\}\mathrm{d}V = \int_B [\mathbf{B}][D]\{E_{ij}\}\mathrm{d}V = \int_B [\mathbf{B}][\mathbf{D}][\overline{\mathbf{B}}]^{\mathrm{T}}\mathrm{d}V \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

Green-Lagrange歪の定義

$$[E_{ij}] = \frac{1}{2}\left(\left[\frac{\partial u_i}{\partial X_j}\right] + \left[\frac{\partial u_j}{\partial X_i}\right] + \left[\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right]\right)$$

$$\begin{aligned}
\mathrm{d}\boldsymbol{x}_1 \cdot \mathrm{d}\boldsymbol{x}_2 &= (\mathbf{F}\mathrm{d}\boldsymbol{X}_1)\cdot(\mathbf{F}\mathrm{d}\boldsymbol{X}_2) \\
&= \mathrm{d}\boldsymbol{X}_1^{\mathrm{T}}\mathbf{U}^{\mathbf{2}}\mathrm{d}\boldsymbol{X}_2 \\
&= \mathrm{d}\boldsymbol{X}_1^{\mathrm{T}}\left(\left[\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right] + \left[\frac{\partial u_j}{\partial X_i}\right] + \left[\frac{\partial u_i}{\partial X_j}\right] + \mathbf{I}\right)\mathrm{d}\boldsymbol{X}_2
\end{aligned}$$



$$E_{11} = \frac{\partial u}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial u}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial v}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial w}{\partial X}$$

$$E_{22} = \frac{\partial v}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial v}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial w}{\partial Y}$$

$$E_{33} = \frac{\partial w}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial u}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial v}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial w}{\partial Z}$$

$$E_{12} = \frac{1}{2}\left(\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X}\frac{\partial v}{\partial Y} + \frac{\partial w}{\partial X}\frac{\partial w}{\partial Y}\right)$$

$$E_{23} = \frac{1}{2}\left(\frac{\partial v}{\partial Z} + \frac{\partial w}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial u}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial v}{\partial Z} + \frac{\partial w}{\partial Y}\frac{\partial w}{\partial Z}\right)$$

$$E_{31} = \frac{1}{2}\left(\frac{\partial w}{\partial X} + \frac{\partial u}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial u}{\partial X} + \frac{\partial v}{\partial Z}\frac{\partial v}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial w}{\partial X}\right)$$

近似解を代入

$$\boxed{\begin{aligned} u &= \boldsymbol{N}\cdot\boldsymbol{U} \\ v &= \boldsymbol{N}\cdot\boldsymbol{V} \\ w &= \boldsymbol{N}\cdot\boldsymbol{W} \end{aligned}}$$

変位の数値解

離散点での各方向の変位ベクトル

・・・回転マトリクス**U**や**V**とは区別

$$\{E_{ij}\} = \begin{Bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{12} \\ 2E_{23} \\ 2E_{31} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial u}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial v}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial w}{\partial X} \\ \frac{\partial v}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial v}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial w}{\partial Y} \\ \frac{\partial w}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial u}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial v}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial w}{\partial Z} \\ \frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X} + \boxed{\frac{\partial u}{\partial X}\frac{\partial u}{\partial Y}} + \frac{\partial v}{\partial X}\frac{\partial v}{\partial Y} + \frac{\partial w}{\partial X}\frac{\partial w}{\partial Y} \\ \frac{\partial v}{\partial Z} + \frac{\partial w}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial u}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial v}{\partial Z} + \frac{\partial w}{\partial Y}\frac{\partial w}{\partial Z} \\ \frac{\partial w}{\partial X} + \frac{\partial u}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial u}{\partial X} + \frac{\partial v}{\partial Z}\frac{\partial v}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial w}{\partial X} \end{Bmatrix} = \begin{bmatrix} \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} \\ \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} \\ \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} \\ \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \left(\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \left(\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \left(\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) \\ \left(\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \left(\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \left(\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) \\ \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \left(\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \left(\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \left(\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

工学的せん断ひずみ

非線形項

$$\frac{\partial u}{\partial X}\frac{\partial u}{\partial Y} = \frac{1}{2}\left(\frac{\partial u}{\partial X}\frac{\partial u}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial u}{\partial X}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X}\right)\boldsymbol{U}$$

未知数をマトリクスに残すことになるが
一旦、そこは諦める（良しとする）

$$\mathbf{B}$$

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \int_B \begin{bmatrix} \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\ \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \\ \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Z} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial X} & \frac{\partial \boldsymbol{N}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial Y} & \frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}}{\partial Z} \end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} \mathrm{d}V$$

形がちょっとだけ違う・・・

同じ形だと、この後の計算の結果、
剛性マトリクスが対称行列になって嬉しい

$$\overline{\mathbf{B}}$$

$$\begin{Bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{12} \\ 2E_{23} \\ 2E_{31} \end{Bmatrix} = \begin{bmatrix} \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} \\ \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} \\ \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} \\ \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{1}{2}\left(\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{1}{2}\left(\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{1}{2}\left(\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{1}{2}\left(\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) \\ \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{1}{2}\left(\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{1}{2}\left(\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

$$[E_{ij}] = \frac{1}{2}\left(\left[\frac{\partial u_i}{\partial X_j}\right] + \left[\frac{\partial u_j}{\partial X_i}\right] + \left[\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right]\right)$$ の**増分形式**は

$$[\Delta E_{ij}] = \frac{1}{2}\left(\left[\frac{\partial \Delta u_i}{\partial X_j}\right] + \left[\frac{\partial \Delta u_j}{\partial X_i}\right] + \left[\frac{\partial \Delta u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right] + \left[\frac{\partial u_k}{\partial X_i}\frac{\partial \Delta u_k}{\partial X_j}\right]\right)$$

$$\begin{Bmatrix} \Delta E_{11} \\ \Delta E_{22} \\ \Delta E_{33} \\ 2\Delta E_{12} \\ 2\Delta E_{23} \\ 2\Delta E_{31} \end{Bmatrix} = \underbrace{\begin{bmatrix}
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} \\
\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} \\
\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \left(\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \left(\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \left(\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) \\
\left(\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \left(\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \left(\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \left(\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \left(\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \left(\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right)
\end{bmatrix}}_{\mathbf{B}} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

構成則を仮定する

$$
\begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & & & \\ \lambda & \lambda + 2\mu & \lambda & & & \\ \lambda & \lambda & \lambda + 2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{bmatrix} \begin{Bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{12} \\ 2E_{23} \\ 2E_{31} \end{Bmatrix}
$$

第2Piola-Kirchhoff応力　　Green-Lagrange歪

現実とは整合しないが簡単

本当はもっと工夫したほうが良いらしい

構成則を仮定する

$$\{S_{ij}\} = [\mathbf{D}]\{E_{ij}\}$$

と書く

現実とは整合しないが簡単

本当はもっと工夫したほうが良いらしい

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \left[\int_B [\mathbf{B}][\mathbf{D}][\overline{\mathbf{B}}]^{\mathrm{T}} \mathrm{d}V \right] \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix} \Rightarrow \boxed{\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \left[\quad \mathbf{K} \quad \right] \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}}$$

剛性マトリクスは逆行列が存在しない

未知と既知（given）で分類

荷重と変位はいずれかが未知の時、対応するもう一方は既知

$$
\begin{Bmatrix} F_X \\ F_Y \\ F_Z \end{Bmatrix} = \begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}
$$

$$
\begin{Bmatrix} \overline{\overline{F}}_X \\ \overline{F}_X \\ \overline{\overline{F}}_Y \\ \overline{F}_Y \\ \overline{\overline{F}}_Z \\ \overline{F}_Z \end{Bmatrix} =
\begin{bmatrix}
\mathbf{K}_{\overline{\overline{F}}_X\overline{\overline{U}}} & \mathbf{K}_{\overline{\overline{F}}_X\overline{U}} & \mathbf{K}_{\overline{\overline{F}}_X\overline{\overline{V}}} & \mathbf{K}_{\overline{\overline{F}}_X\overline{V}} & \mathbf{K}_{\overline{\overline{F}}_X\overline{\overline{W}}} & \mathbf{K}_{\overline{\overline{F}}_X\overline{W}} \\
\mathbf{K}_{\overline{F}_X\overline{\overline{U}}} & \mathbf{K}_{\overline{F}_X\overline{U}} & \mathbf{K}_{\overline{F}_X\overline{\overline{V}}} & \mathbf{K}_{\overline{F}_X\overline{V}} & \mathbf{K}_{\overline{F}_X\overline{\overline{W}}} & \mathbf{K}_{\overline{F}_X\overline{W}} \\
\mathbf{K}_{\overline{\overline{F}}_Y\overline{\overline{U}}} & \mathbf{K}_{\overline{\overline{F}}_Y\overline{U}} & \mathbf{K}_{\overline{\overline{F}}_Y\overline{\overline{V}}} & \mathbf{K}_{\overline{\overline{F}}_Y\overline{V}} & \mathbf{K}_{\overline{\overline{F}}_Y\overline{\overline{W}}} & \mathbf{K}_{\overline{\overline{F}}_Y\overline{W}} \\
\mathbf{K}_{\overline{F}_Y\overline{\overline{U}}} & \mathbf{K}_{\overline{F}_Y\overline{U}} & \mathbf{K}_{\overline{F}_Y\overline{\overline{V}}} & \mathbf{K}_{\overline{F}_Y\overline{V}} & \mathbf{K}_{\overline{F}_Y\overline{\overline{W}}} & \mathbf{K}_{\overline{F}_Y\overline{W}} \\
\mathbf{K}_{\overline{\overline{F}}_Z\overline{\overline{U}}} & \mathbf{K}_{\overline{\overline{F}}_Z\overline{U}} & \mathbf{K}_{\overline{\overline{F}}_Z\overline{\overline{V}}} & \mathbf{K}_{\overline{\overline{F}}_Z\overline{V}} & \mathbf{K}_{\overline{\overline{F}}_Z\overline{\overline{W}}} & \mathbf{K}_{\overline{\overline{F}}_Z\overline{W}} \\
\mathbf{K}_{\overline{F}_Z\overline{\overline{U}}} & \mathbf{K}_{\overline{F}_Z\overline{U}} & \mathbf{K}_{\overline{F}_Z\overline{\overline{V}}} & \mathbf{K}_{\overline{F}_Z\overline{V}} & \mathbf{K}_{\overline{F}_Z\overline{\overline{W}}} & \mathbf{K}_{\overline{F}_Z\overline{W}}
\end{bmatrix}
\begin{Bmatrix} \overline{\overline{U}} \\ \overline{U} \\ \overline{\overline{V}} \\ \overline{V} \\ \overline{\overline{W}} \\ \overline{W} \end{Bmatrix}
$$

荷重が未知のものは消す

$$
\begin{Bmatrix} \bar{\bar{F}}_X \\ \bar{F}_X \\ \bar{\bar{F}}_Y \\ \bar{F}_Y \\ \bar{\bar{F}}_Z \\ \bar{F}_Z \end{Bmatrix}
=
\begin{bmatrix}
\mathbf{K}_{\bar{\bar{F}}_X\bar{\bar{U}}} & \mathbf{K}_{\bar{\bar{F}}_X\bar{U}} & \mathbf{K}_{\bar{\bar{F}}_X\bar{\bar{V}}} & \mathbf{K}_{\bar{\bar{F}}_X\bar{V}} & \mathbf{K}_{\bar{\bar{F}}_X\bar{\bar{W}}} & \mathbf{K}_{\bar{\bar{F}}_X\bar{W}} \\
\mathbf{K}_{\bar{F}_X\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_X\bar{U}} & \mathbf{K}_{\bar{F}_X\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_X\bar{V}} & \mathbf{K}_{\bar{F}_X\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_X\bar{W}} \\
\mathbf{K}_{\bar{\bar{F}}_Y\bar{\bar{U}}} & \mathbf{K}_{\bar{\bar{F}}_Y\bar{U}} & \mathbf{K}_{\bar{\bar{F}}_Y\bar{\bar{V}}} & \mathbf{K}_{\bar{\bar{F}}_Y\bar{V}} & \mathbf{K}_{\bar{\bar{F}}_Y\bar{\bar{W}}} & \mathbf{K}_{\bar{\bar{F}}_Y\bar{W}} \\
\mathbf{K}_{\bar{F}_Y\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_Y\bar{U}} & \mathbf{K}_{\bar{F}_Y\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_Y\bar{V}} & \mathbf{K}_{\bar{F}_Y\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_Y\bar{W}} \\
\mathbf{K}_{\bar{\bar{F}}_Z\bar{\bar{U}}} & \mathbf{K}_{\bar{\bar{F}}_Z\bar{U}} & \mathbf{K}_{\bar{\bar{F}}_Z\bar{\bar{V}}} & \mathbf{K}_{\bar{\bar{F}}_Z\bar{V}} & \mathbf{K}_{\bar{\bar{F}}_Z\bar{\bar{W}}} & \mathbf{K}_{\bar{\bar{F}}_Z\bar{W}} \\
\mathbf{K}_{\bar{F}_Z\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_Z\bar{U}} & \mathbf{K}_{\bar{F}_Z\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_Z\bar{V}} & \mathbf{K}_{\bar{F}_Z\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_Z\bar{W}}
\end{bmatrix}
\begin{Bmatrix} \bar{\bar{U}} \\ \bar{U} \\ \bar{\bar{V}} \\ \bar{V} \\ \bar{\bar{W}} \\ \bar{W} \end{Bmatrix}
$$

$$\Downarrow$$

$$
\begin{Bmatrix} \bar{F}_X \\ \bar{F}_Y \\ \bar{F}_Z \end{Bmatrix}
=
\begin{bmatrix}
\mathbf{K}_{\bar{F}_X\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_X\bar{U}} & \mathbf{K}_{\bar{F}_X\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_X\bar{V}} & \mathbf{K}_{\bar{F}_X\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_X\bar{W}} \\
\mathbf{K}_{\bar{F}_Y\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_Y\bar{U}} & \mathbf{K}_{\bar{F}_Y\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_Y\bar{V}} & \mathbf{K}_{\bar{F}_Y\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_Y\bar{W}} \\
\mathbf{K}_{\bar{F}_Z\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_Z\bar{U}} & \mathbf{K}_{\bar{F}_Z\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_Z\bar{V}} & \mathbf{K}_{\bar{F}_Z\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_Z\bar{W}}
\end{bmatrix}
\begin{Bmatrix} \bar{\bar{U}} \\ \bar{U} \\ \bar{\bar{V}} \\ \bar{V} \\ \bar{\bar{W}} \\ \bar{W} \end{Bmatrix}
$$

変位が既知のものは移項する

$$
\begin{Bmatrix} \bar{F}_X \\ \bar{F}_Y \\ \bar{F}_Z \end{Bmatrix}
=
\begin{bmatrix}
\mathbf{K}_{\bar{F}_X\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_X\bar{U}} & \mathbf{K}_{\bar{F}_X\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_X\bar{V}} & \mathbf{K}_{\bar{F}_X\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_X\bar{W}} \\
\mathbf{K}_{\bar{F}_Y\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_Y\bar{U}} & \mathbf{K}_{\bar{F}_Y\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_Y\bar{V}} & \mathbf{K}_{\bar{F}_Y\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_Y\bar{W}} \\
\mathbf{K}_{\bar{F}_Z\bar{\bar{U}}} & \mathbf{K}_{\bar{F}_Z\bar{U}} & \mathbf{K}_{\bar{F}_Z\bar{\bar{V}}} & \mathbf{K}_{\bar{F}_Z\bar{V}} & \mathbf{K}_{\bar{F}_Z\bar{\bar{W}}} & \mathbf{K}_{\bar{F}_Z\bar{W}}
\end{bmatrix}
\begin{Bmatrix} \bar{\bar{U}} \\ \bar{U} \\ \bar{\bar{V}} \\ \bar{V} \\ \bar{\bar{W}} \\ \bar{W} \end{Bmatrix}
$$

$$\Downarrow$$

$$
\begin{Bmatrix}
\bar{F}_X - \mathbf{K}_{\bar{F}_X\bar{\bar{U}}}\bar{\bar{U}} - \mathbf{K}_{\bar{F}_X\bar{\bar{V}}}\bar{\bar{V}} - \mathbf{K}_{\bar{F}_X\bar{\bar{W}}}\bar{\bar{W}} \\
\bar{F}_Y - \mathbf{K}_{\bar{F}_Y\bar{\bar{U}}}\bar{\bar{U}} - \mathbf{K}_{\bar{F}_Y\bar{\bar{V}}}\bar{\bar{V}} - \mathbf{K}_{\bar{F}_Y\bar{\bar{W}}}\bar{\bar{W}} \\
\bar{F}_Z - \mathbf{K}_{\bar{F}_Z\bar{\bar{U}}}\bar{\bar{U}} - \mathbf{K}_{\bar{F}_Z\bar{\bar{V}}}\bar{\bar{V}} - \mathbf{K}_{\bar{F}_Z\bar{\bar{W}}}\bar{\bar{W}}
\end{Bmatrix}
=
\begin{bmatrix}
\mathbf{K}_{\bar{F}_X\bar{U}} & \mathbf{K}_{\bar{F}_X\bar{V}} & \mathbf{K}_{\bar{F}_X\bar{W}} \\
\mathbf{K}_{\bar{F}_Y\bar{U}} & \mathbf{K}_{\bar{F}_Y\bar{V}} & \mathbf{K}_{\bar{F}_Y\bar{W}} \\
\mathbf{K}_{\bar{F}_Z\bar{U}} & \mathbf{K}_{\bar{F}_Z\bar{V}} & \mathbf{K}_{\bar{F}_Z\bar{W}}
\end{bmatrix}
\begin{Bmatrix} \bar{U} \\ \bar{V} \\ \bar{W} \end{Bmatrix}
$$

定義を更新する

$$
\begin{Bmatrix}
\bar{F}_X - \mathbf{K}_{\bar{F}_X\bar{\bar{U}}}\bar{\bar{U}} - \mathbf{K}_{\bar{F}_X\bar{\bar{V}}}\bar{\bar{V}} - \mathbf{K}_{\bar{F}_X\bar{\bar{W}}}\bar{\bar{W}} \\
\bar{F}_Y - \mathbf{K}_{\bar{F}_Y\bar{\bar{U}}}\bar{\bar{U}} - \mathbf{K}_{\bar{F}_Y\bar{\bar{V}}}\bar{\bar{V}} - \mathbf{K}_{\bar{F}_Y\bar{\bar{W}}}\bar{\bar{W}} \\
\bar{F}_Z - \mathbf{K}_{\bar{F}_Z\bar{\bar{U}}}\bar{\bar{U}} - \mathbf{K}_{\bar{F}_Z\bar{\bar{V}}}\bar{\bar{V}} - \mathbf{K}_{\bar{F}_Z\bar{\bar{W}}}\bar{\bar{W}}
\end{Bmatrix}
=
\begin{bmatrix}
\mathbf{K}_{\bar{F}_X\bar{U}} & \mathbf{K}_{\bar{F}_X\bar{V}} & \mathbf{K}_{\bar{F}_X\bar{W}} \\
\mathbf{K}_{\bar{F}_Y\bar{U}} & \mathbf{K}_{\bar{F}_Y\bar{V}} & \mathbf{K}_{\bar{F}_Y\bar{W}} \\
\mathbf{K}_{\bar{F}_Z\bar{U}} & \mathbf{K}_{\bar{F}_Z\bar{V}} & \mathbf{K}_{\bar{F}_Z\bar{W}}
\end{bmatrix}
\begin{Bmatrix}
\bar{U} \\ \bar{V} \\ \bar{W}
\end{Bmatrix}
$$

$$\Downarrow$$

$$
\begin{Bmatrix}
\hat{F}_X \\ \hat{F}_Y \\ \hat{F}_Z
\end{Bmatrix}
=
\begin{bmatrix}
\bar{\mathbf{K}}
\end{bmatrix}
\begin{Bmatrix}
\bar{U} \\ \bar{V} \\ \bar{W}
\end{Bmatrix}
$$

境界条件を導入することで**逆行列**が存在するようになった！

$$\begin{Bmatrix} \boldsymbol{F}_X \\ \boldsymbol{F}_Y \\ \boldsymbol{F}_Z \end{Bmatrix} = \begin{bmatrix} & & \\ & \mathbf{K} & \\ & & \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

力のつりあい式

境界条件を代入

$$\begin{Bmatrix} \widehat{\boldsymbol{F}}_X \\ \widehat{\boldsymbol{F}}_Y \\ \widehat{\boldsymbol{F}}_Z \end{Bmatrix} = \begin{Bmatrix} \overline{\boldsymbol{F}}_X - \mathbf{K}_{\overline{F}_X \overline{\overline{U}}} \overline{\overline{\boldsymbol{U}}} - \mathbf{K}_{\overline{F}_X \overline{\overline{V}}} \overline{\overline{\boldsymbol{V}}} - \mathbf{K}_{\overline{F}_X \overline{\overline{W}}} \overline{\overline{\boldsymbol{W}}} \\ \overline{\boldsymbol{F}}_Y - \mathbf{K}_{\overline{F}_Y \overline{\overline{U}}} \overline{\overline{\boldsymbol{U}}} - \mathbf{K}_{\overline{F}_Y \overline{\overline{V}}} \overline{\overline{\boldsymbol{V}}} - \mathbf{K}_{\overline{F}_Y \overline{\overline{W}}} \overline{\overline{\boldsymbol{W}}} \\ \overline{\boldsymbol{F}}_Z - \mathbf{K}_{\overline{F}_Z \overline{\overline{U}}} \overline{\overline{\boldsymbol{U}}} - \mathbf{K}_{\overline{F}_Z \overline{\overline{V}}} \overline{\overline{\boldsymbol{V}}} - \mathbf{K}_{\overline{F}_Z \overline{\overline{W}}} \overline{\overline{\boldsymbol{W}}} \end{Bmatrix}$$

$$\begin{Bmatrix} \overline{\boldsymbol{U}} \\ \overline{\boldsymbol{V}} \\ \overline{\boldsymbol{W}} \end{Bmatrix} = \begin{bmatrix} & & \\ & \overline{\mathbf{K}} & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} \widehat{\boldsymbol{F}}_X \\ \widehat{\boldsymbol{F}}_Y \\ \widehat{\boldsymbol{F}}_Z \end{Bmatrix}$$

未知変位を計算

構成則を仮定

$$\begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix} = \boxed{\begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & & & \\ \lambda & \lambda + 2\mu & \lambda & & & \\ \lambda & \lambda & \lambda + 2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{bmatrix}} \begin{Bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{12} \\ 2E_{23} \\ 2E_{31} \end{Bmatrix}$$

剛性マトリクス

最初は非線形項を無視
（計算できないので）

$$[\mathbf{K}] = \int_B [\mathbf{B}][\mathbf{D}][\overline{\mathbf{B}}]^{\mathrm{T}} \mathrm{d}V$$

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} \boldsymbol{N}^{\mathrm{T}} & & \\ & \boldsymbol{N}^{\mathrm{T}} & \\ & & \boldsymbol{N}^{\mathrm{T}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \\ \boldsymbol{W} \end{Bmatrix}$$

積分点位置の
変位分布を更新

$$\begin{bmatrix}
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} \\
\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} \\
\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \left(\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \left(\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \left(\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) \\
\left(\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \left(\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \left(\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \left(\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \left(\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \left(\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right)
\end{bmatrix}$$

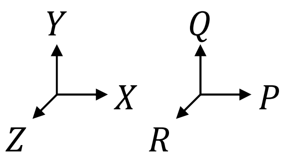
$$\begin{bmatrix}
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} & \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} \\
\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} & \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} \\
\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{1}{2}\left(\frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{1}{2}\left(\frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X}\right) \\
\frac{1}{2}\left(\frac{\partial u}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{1}{2}\left(\frac{\partial v}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y} + \frac{1}{2}\left(\frac{\partial w}{\partial Y}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Y}\right) \\
\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z} + \frac{1}{2}\left(\frac{\partial u}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial u}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right) & \frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{1}{2}\left(\frac{\partial w}{\partial Z}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial X} + \frac{\partial w}{\partial X}\frac{\partial \boldsymbol{N}^{\mathrm{T}}}{\partial Z}\right)
\end{bmatrix}$$

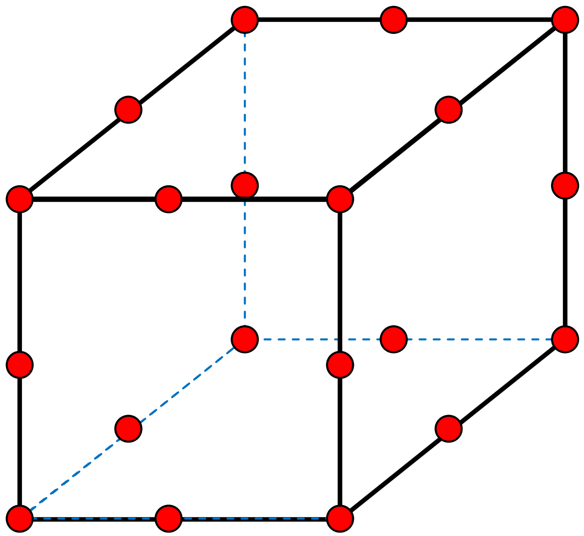
繰り返して収束計算

形状関数：$\boldsymbol{N}^{\mathrm{T}}$

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \Rightarrow \begin{Bmatrix} P \\ Q \\ R \end{Bmatrix}$$

初期配置の座標系　　要素内の局所座標





各辺が-1から1の範囲に定義される
立方体になるように座標変換

形状関数：$\boldsymbol{N}^{\mathrm{T}}$

$$N_3 = -\frac{1}{8}(1-P)(1+Q)(1-R)(2+P-Q+R)$$

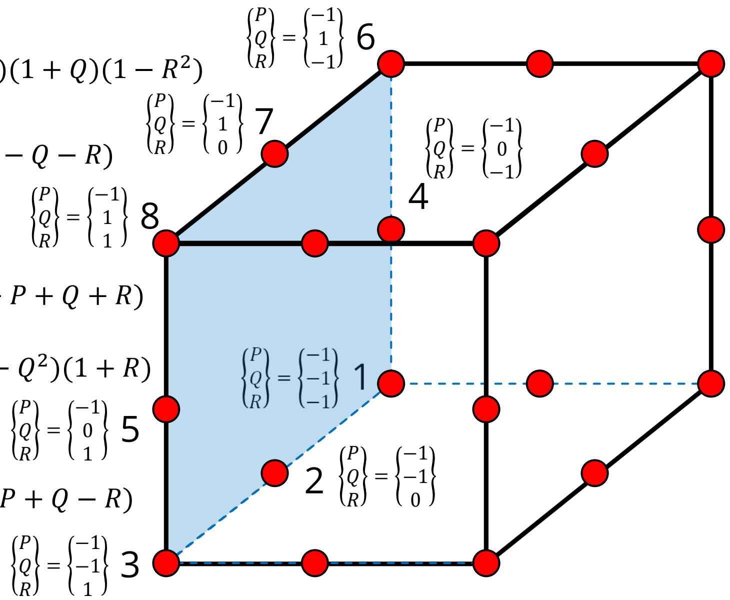
$$N_5 = \frac{1}{4}(1-P)(1+Q)(1-R^2)$$

$$N_5 = -\frac{1}{8}(1-P)(1+Q)(1+R)(2+P-Q-R)$$

$$N_1 = -\frac{1}{8}(1-P)(1-Q)(1-R)(2+P+Q+R)$$

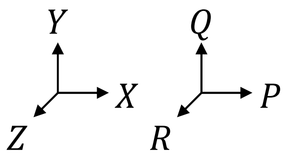
$$N_5 = \frac{1}{4}(1-P)(1-Q^2)(1+R)$$

$$N_3 = -\frac{1}{8}(1-P)(1-Q)(1+R)(2+P+Q-R)$$



$$N_4 = \frac{1}{4}(1-P)(1-Q^2)(1-R)$$

$$N_2 = \frac{1}{4}(1-P)(1-Q)(1-R^2)$$



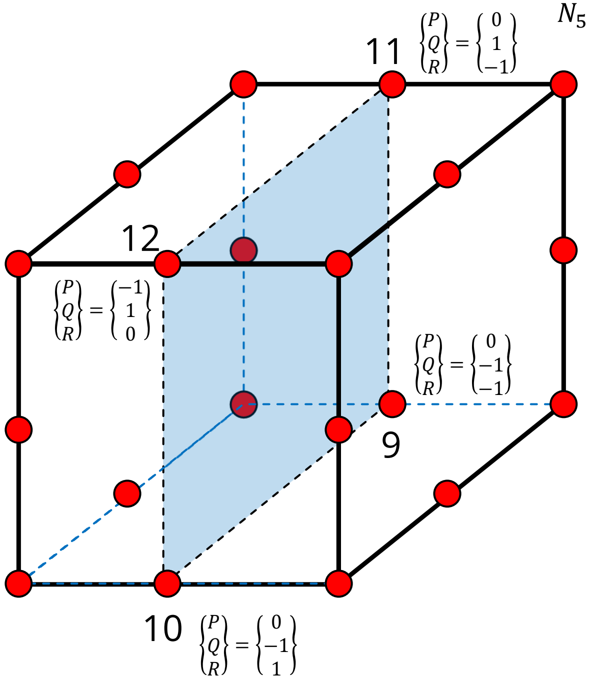

```
N(1) = -1/8*(1-X)*(1-Y)*(1-Z)*(2+X+Y+Z);
N(2) =  1/4*(1-Z^2)*(1-X)*(1-Y);
N(3) = -1/8*(1-X)*(1-Y)*(1+Z)*(2+X+Y-Z);
N(4) =  1/4*(1-Y^2)*(1-X)*(1-Z);
N(5) =  1/4*(1-Y^2)*(1-X)*(1+Z);
N(6) = -1/8*(1-X)*(1+Y)*(1-Z)*(2+X-Y+Z);
N(7) =  1/4*(1-Z^2)*(1-X)*(1+Y);
N(8) = -1/8*(1-X)*(1+Y)*(1+Z)*(2+X-Y-Z);
```

形状関数：$\boldsymbol{N}^{\mathrm{T}}$

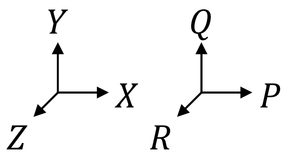
```
N(1) = -1/8*(1-X)*(1-Y)*(1-Z)*(2+X+Y+Z);
N(2) =  1/4*(1-Z^2)*(1-X)*(1-Y);
N(3) = -1/8*(1-X)*(1-Y)*(1+Z)*(2+X+Y-Z);
N(4) =  1/4*(1-Y^2)*(1-X)*(1-Z);
N(5) =  1/4*(1-Y^2)*(1-X)*(1+Z);
N(6) = -1/8*(1-X)*(1+Y)*(1-Z)*(2+X-Y+Z);
N(7) =  1/4*(1-Z^2)*(1-X)*(1+Y);
N(8) = -1/8*(1-X)*(1+Y)*(1+Z)*(2+X-Y-Z);

N(9) =  1/4*(1-X^2)*(1-Y)*(1-Z);
N(10)=  1/4*(1-X^2)*(1-Y)*(1+Z);
N(11)=  1/4*(1-X^2)*(1+Y)*(1-Z);
N(12)=  1/4*(1-X^2)*(1+Y)*(1+Z);
```

$$N_5 = \frac{1}{4}(1-P^2)(1+Q)(1-R)$$

$$N_{12} = \frac{1}{4}(1-P^2)(1+Q)(1+R)$$



$$N_9 = \frac{1}{4}(1-P^2)(1-Q)(1-R)$$



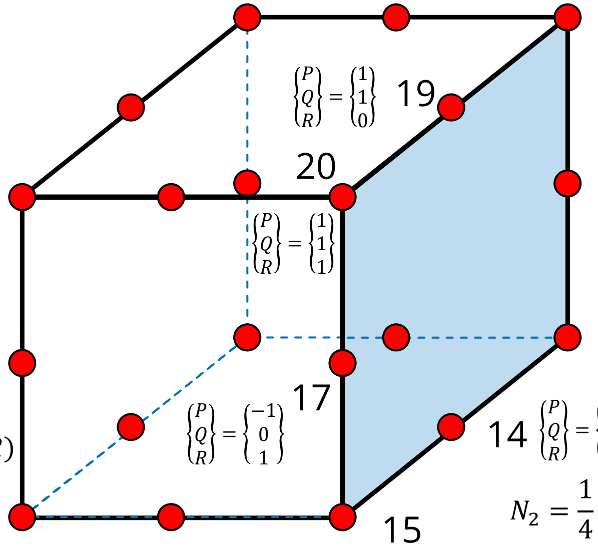
$$N_4 = \frac{1}{4}(1-P^2)(1-Q)(1+R)$$

形状関数：$\boldsymbol{N}^{\mathrm{T}}$

```
N(1) = -1/8*(1-X)*(1-Y)*(1-Z)*(2+X+Y+Z);
N(2) =  1/4*(1-Z^2)*(1-X)*(1-Y);
N(3) = -1/8*(1-X)*(1-Y)*(1+Z)*(2+X+Y-Z);
N(4) =  1/4*(1-Y^2)*(1-X)*(1-Z);
N(5) =  1/4*(1-Y^2)*(1-X)*(1+Z);
N(6) = -1/8*(1-X)*(1+Y)*(1-Z)*(2+X-Y+Z);
N(7) =  1/4*(1-Z^2)*(1-X)*(1+Y);
N(8) = -1/8*(1-X)*(1+Y)*(1+Z)*(2+X-Y-Z);

N(9) =  1/4*(1-X^2)*(1-Y)*(1-Z);
N(10)=  1/4*(1-X^2)*(1-Y)*(1+Z);
N(11)=  1/4*(1-X^2)*(1+Y)*(1-Z);
N(12)=  1/4*(1-X^2)*(1+Y)*(1+Z);

N(13)= -1/8*(1+X)*(1-Y)*(1-Z)*(2-X+Y+Z);
N(14)=  1/4*(1-Z^2)*(1+X)*(1-Y);
N(15)= -1/8*(1+X)*(1-Y)*(1+Z)*(2-X+Y-Z);
N(16)=  1/4*(1-Y^2)*(1+X)*(1-Z);
N(17)=  1/4*(1-Y^2)*(1+X)*(1+Z);
N(18)= -1/8*(1+X)*(1+Y)*(1-Z)*(2-X-Y+Z);
N(19)=  1/4*(1-Z^2)*(1+X)*(1+Y);
N(20)= -1/8*(1+X)*(1+Y)*(1+Z)*(2-X-Y-Z);
```

$$N_{18} = -\frac{1}{8}(1+P)(1+Q)(1-R)(2-P-Q+R)$$

$$N_{19} = \frac{1}{4}(1+P)(1+Q)(1-R^2)$$



$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ -1 \end{Bmatrix}$ (18)

$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$ (19)

$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$ (16)

$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$ (20)

$$N_{16} = \frac{1}{4}(1+P)(1-Q^2)(1-R)$$

$$N_{13} = -\frac{1}{8}(1+P)(1-Q)(1-R)(2-P+Q+R)$$

$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix}$ (13)

$$N_{20} = -\frac{1}{8}(1+P)(1+Q)(1+R)(2-P-Q-R)$$

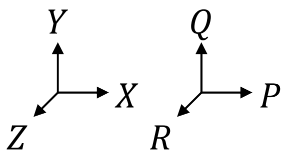
$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix}$ (17)

$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \\ 0 \end{Bmatrix}$ (14)

$$N_{17} = \frac{1}{4}(1+P)(1-Q^2)(1+R)$$

$$N_2 = \frac{1}{4}(1+P)(1-Q)(1-R^2)$$

$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \\ 1 \end{Bmatrix}$ (15)



$$N_{15} = -\frac{1}{8}(1+P)(1-Q)(1+R)(2-P+Q-R)$$

1次要素を使うと**ロッキング現象**が発生

（変形しない）

（対策）

・**高次要素**の利用

・低減積分の適用

大変形問題でも解ける