日本建築学会大会学術講演梗概集
（関東） 2015 年 9 月

Crack Width Evaluation of RC Members Reinforced with Braided AFRP Bars
(Part 2: Model for Bond Constitutive law and Crack Width Prediction Method)

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Crack width Bond stress Slippage
AFRP bars Tri-linear Evaluation

1. Introduction

In part 1, both pullout bond test and tensile bond test were carried out to obtain the bond constitutive law and measured crack width. In this part, tri-linear models of bond constitutive law both for normal surface type and sand-coated type AFRP bars are proposed. Using the tri-linear models, solving an equation of the relationship between reinforcement strain and slip at the loaded end[1)] can obtain crack width prediction formulas for RC members reinforced with AFRP bars.

2. Tri-linear Model

As there is a large difference between the bond behaviors of two types of AFRP surface, two types of tri-linear models are introduced. One is for the normal surface type (RA7, RA13) and the other is for the sand-coated type (RA7S, RA13S).

2.1 Tri-linear model for normal surface type

Fig. 1 shows the tri-linear model for normal surface type. The definitions and mathematical expressions are given as follows.

τ_{max} = maximum bond stress, $s_{\max}$ = slip at $\tau_{\max}$

$\tau_2 = 2/3 \cdot \tau_{\max}$, s_2 = slip at τ_2, $\tau_1 = k_1 \cdot s_1$

$$s_1 = \frac{6 \cdot G_{f2} - 2 \cdot s_2 \cdot \tau_{\max}}{3 \cdot k_1 \cdot s_2 - 2 \cdot \tau_{\max}}$$

G_{f2} = fracture energy during bond stress from 0 to τ_2

k_1 = initial stiffness, $k_2 = \dfrac{\tau_2 - \tau_1}{s_2 - s_1}$, $k_3 = \dfrac{\tau_{\max} - \tau_2}{s_{\max} - s_2}$

The maximum bond stress of the RA13 specimens was not obtained due to concrete splitting. Those values are assumed as the average value of RA7 specimens.

2.2 Tri-linear model for sand-coated type

Tri-linear model for sand-coated type is shown in Fig. 2. The definitions and mathematical expressions are given as follows.

τ_u = minimum value after slipping bond stress, s_u = slip at τ_u

$\tau_{\max}$ = slipping bond stress , $s_{\max}$ = slip at $\tau_{\max}$, $\tau_1 = k_1 \cdot s_1$

$$s_1 = \frac{2 \cdot G_{f2} - s_{\max} \cdot \tau_{\max}}{k_1 \cdot s_{\max} - \tau_{\max}}$$

G_{f2} = fracture energy during bond stress from 0 to τ_2

k_1 = initial stiffness, $k_2 = \dfrac{\tau_2 - \tau_1}{s_2 - s_1}$, $k_3 = \dfrac{\tau_u - \tau_{\max}}{s_u - s_{\max}}$

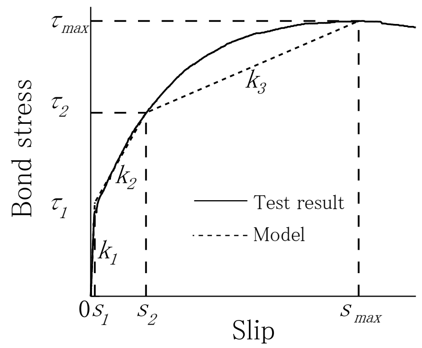

Fig.1. Tri-linear model for normal surface type

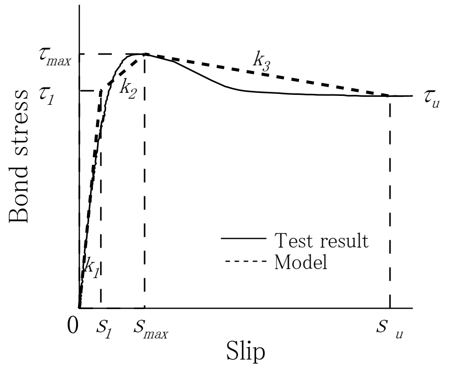

Fig.2. Tri-linear model for sand adhesion type

$$w_{cr} = 2 \cdot \sqrt{\frac{2\sigma_{ct} A_c}{k_1 \phi_b} \left(\varepsilon_l - \frac{1+np}{2} \cdot \frac{\sigma_{ct} A_c}{E_b A_b} \right)} \quad (1)$$

$$(w_{cr} < 2 \cdot s_1)$$

$$w_{cr} = 2 \cdot \left\{ (1 - \frac{k_1}{k_2}) \cdot s_1 + \sqrt{\frac{2\sigma_{ct} A_c}{k_2 \phi_b} \left(\varepsilon_l - \frac{1+np}{2} \cdot \frac{\sigma_{ct} A_c}{E_b A_b} \right) + s_1^{\,2} \cdot \frac{k_1 \cdot (k_1 - k_2)}{k_2^{\,2}}} \right\} \quad (2)$$

$$(k_2 \neq 0, 2 \cdot s_1 < w_{cr} < 2 \cdot s_2, w_{cr} > 2 \cdot (1 - \frac{k_1}{k_2}) \cdot s_1)$$

$$w_{cr} = 2 \cdot \left\{ (1 - \frac{k_1}{k_2}) \cdot s_1 - \sqrt{\frac{2\sigma_{ct} A_c}{k_2 \phi_b} \left(\varepsilon_l - \frac{1+np}{2} \cdot \frac{\sigma_{ct} A_c}{E_b A_b} \right) + s_1^{\,2} \cdot \frac{k_1 \cdot (k_1 - k_2)}{k_2^{\,2}}} \right\} \quad (3)$$

$$(k_2 \neq 0, 2 \cdot s_1 < w_{cr} < 2 \cdot s_2, w_{cr} < 2 \cdot (1 - \frac{k_1}{k_2}) \cdot s_1)$$

$$w_{cr} = 2 \cdot \left\{ \frac{1}{2} \cdot s_1 + \frac{\sigma_{ct} A_c}{k_1 \phi_b s_1} \left(\varepsilon_l - \frac{1+np}{2} \cdot \frac{\sigma_{ct} A_c}{E_b A_b} \right) \right\} \quad (4)$$

$$(k_2 = 0, 2 \cdot s_1 < w_{cr} < 2 \cdot s_2)$$

The stiff resistance is provided by bearing between sand and concrete in the case of the sand-coated surface of bars. When this resistance fails, the bar starts slipping with a sudden fall of bond stress. The bond stress at where the bar starts slipping is called “slipping bond stress”.

3. Theoretical Calculation Formulas

The following equation gives the relationship between reinforcement strain and slip at the loaded end[1].

$$\varepsilon_{bl} = \frac{\phi_b}{\sigma_{ct} \cdot A_c} \int_0^{s_l} \tau_b \cdot ds + \left(\frac{1+n \cdot p}{2}\right) \frac{\sigma_{ct} \cdot A_c}{E_b \cdot A_b}$$

where,

ε_{bl} : strain of reinforcement at loaded end

ϕ_b : perimeter of bar, σ_{ct} : tensile strength of concrete

A_c : sectional area of concrete, A_b : sectional area of bar

E_b : elastic modulus of reinforcement, s_l : slip at loaded end

E_c : elastic modulus of concrete, τ_b : bond stress

n : elastic modulus ratio $= E_b/E_c$

p : reinforcement ratio $= A_b/A_c$

Using the tri-linear models and an assumption that the slip at the loaded end gives a half of crack width, theoretical calculation formulas for crack width Eq.(1)~Eq.(7) can be obtained.

$$w_{cr} = 2 \cdot \left\{ s_2 - \frac{\tau_2}{k_3} + \sqrt{\frac{2\sigma_{ct}A_c}{k_3\phi_b}\left(\varepsilon_l - \frac{1+np}{2}\cdot\frac{\sigma_{ct}A_c}{E_bA_b}\right) + \frac{\tau_2^{\,2}}{k_3^{\,2}} - \frac{k_1s_1s_2 + \tau_2s_2 - \tau_2s_1}{k_3}} \right\} \quad (5)$$

$$(k_3 \neq 0, w_{cr} > 2 \cdot s_2, w_{cr} > 2 \cdot (1 - \frac{k_2}{k_3}) \cdot s_2)$$

$$w_{cr} = 2 \cdot \left\{ s_2 - \frac{\tau_2}{k_3} - \sqrt{\frac{2\sigma_{ct}A_c}{k_3\phi_b}\left(\varepsilon_l - \frac{1+np}{2}\cdot\frac{\sigma_{ct}A_c}{E_bA_b}\right) + \frac{\tau_2^{\,2}}{k_3^{\,2}} - \frac{k_1s_1s_2 + \tau_2s_2 - \tau_2s_1}{k_3}} \right\} \quad (6)$$

$$(k_3 \neq 0, w_{cr} > 2 \cdot s_2, w_{cr} < 2(1 - \frac{k_2}{k_3}) \cdot s_2)$$

$$w_{cr} = 2 \cdot \left\{ \frac{2\sigma_{ct}A_c}{\phi_b\tau_2}\left(\varepsilon_l - \frac{1+np}{2}\cdot\frac{\sigma_{ct}A_c}{E_bA_b}\right) + \frac{1}{2}\cdot s_2 + \frac{1}{2}s_1\cdot(1 - \frac{k_1s_2}{\tau_2}) \right\} \quad (7)$$

$$(k_3 = 0, w_{cr} > 2 \cdot s_2)$$

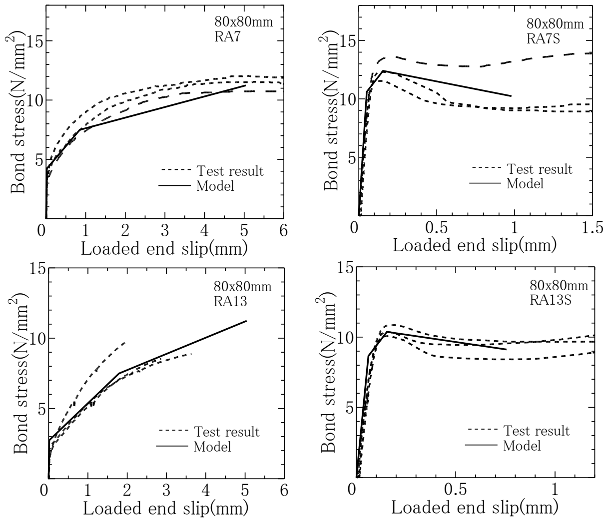

Fig.3. Models of bond constitutive law

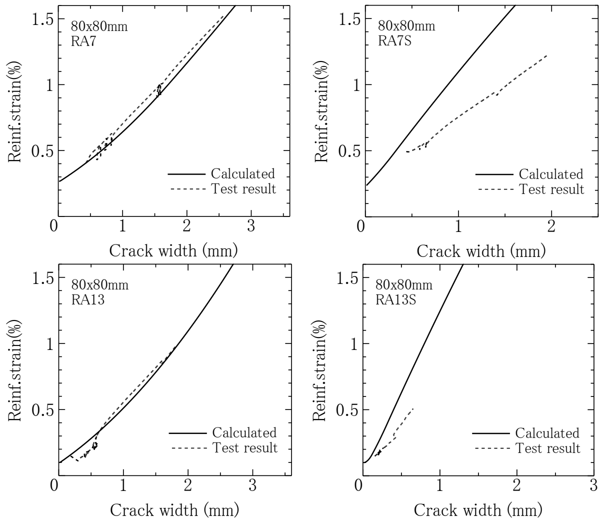

Fig.4.Crack width – reinforcement strain relationship

4. Adaptability of Proposed Method

Examples of the models of bond constitutive law are shown in Fig.3. The characteristic points of the models are obtained by the average values of three specimens according to the definitions of tri-linear models. Examples of relationships between reinforcement strain and measured crack width are shown in Fig.4 for specimens of 80 x 80mm, comparing experimental results and calculated curves by the proposed formulas. The experimental curves of the specimens reinforced with normal surface type (RA7 or RA13) show good agreements with calculated ones. However, calculated curves for the sand-coated type (RA7S or RA13S) show a little deviation from the experimental curves. It is considered that perpendicular cracks in tensile bond test affect the experimental measured crack width in those specimens.

5. Conclusions

Tri-linear models for bond constitutive law are proposed based on the experimental results of pullout bond test both for normal surface type and sand-coated type AFRP bars. Based on the bond theory which gives the relationship between reinforcement strain and slip at the loaded, theoretical calculation formulas to predict crack width of RC members for AFRP bars are proposed. The predicted crack widths show a good agreement with test results.

References

1) Kanakubo T., Yamato N., Crack Width Prediction Method for Steel and FRP Reinforcement Based on Bond Theory, Journal of Advanced Concrete Technology, Vol. 12(2014), No. 9, pp. 310-319

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