Development of Seismic Damage Finite Element Code for Reinforced Concrete Framed Structures

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ABSTRACT

A new finite element code using the Adaptively Shifted Integration (ASI) technique with a linear Timoshenko beam element, which can express member fracture by a plastic hinge located at an exact position with a simultaneous release of the resultant forces in the element, is applied to the seismic damage analysis of reinforced concrete (RC) framed structures. Contact between members is also considered in order to obtain results that agree more closely with actual behaviors such as intermediate-layer failure. By using the proposed code, sufficiently reliable solutions have been obtained and the results reveal that this technique can be used in the numerical estimation of structural reliabilities.

INTRODUCTION

In the conventional design of a building, only static analysis in the horizontal and uniaxial directions is commonly carried out, in order to minimize calculation costs. This approach may ensure the structural strength of the building, if there is sufficient strength to support the load in vertical direction. However, the mass system model replaces the building layer in dynamic analysis, and the complicated dynamic behavior of the structure at member level is not sufficiently examined. Therefore, the development of a more precise and more efficient dynamic analysis code is strongly desired. Recently, significant advances in the field of computers have been removing the calculation cost restrictions, and various dynamic analysis codes are being developed. In this study, the Adaptively Shifted Integration (ASI) technique (Toi and Isobe 1993) is implemented into the finite element code in order to develop a more precise and less calculation-time-consuming seismic damage analytical tool.

The purpose of this study is to verify the validity of the ASI technique in seismic damage analysis and to construct a highly efficient structural design tool for RC structures. Simple

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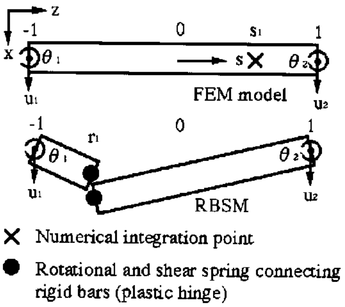


Fig.1 Linear Timoshenko beam element and its physical equivalent

numerical tests showing the validity of the modeling of an RC beam along with the validity of the implemented scheme, are carried out. Also, those examples involving structural discontinuities such as member fracture are carried out to show the expansibility of the scheme. A contact algorithm is added to the code to reproduce phenomena such as intermediate-layer failure.

ADAPTIVELY SHIFTED INTEGRATION TECHNIQUE

A relation between the location of a numerical integration point (s_1) and occurrence of a plastic hinge (r_1) in the linear Timoshenko beam element ($-1 \leqq r_1, s_1 \leqq 1$) is obtained by considering the equivalence conditions between the strain energy approximations of a linear Timoshenko beam element and a physical model known as the rigid-bodies spring model (RBSM). Referring to Figure 1, the relation is expressed by the following equation (Toi 1991):

$$s_1 = -r_1 \ or \ r_1 = -s_1 \tag{1}$$

where s_1 and r_1 are position of the numerical integration point and position of the plastic hinge or member fracture, respectively.

When the entire region in an element behaves elastically, the midpoint of the element (s_1=0) is the most appropriate integration point from considerations of accuracy and symmetry. The internal force vector at step *n* based on the updated Lagrangian formulation, is expressed as

$$\{{}^n_nF\} = \int_{^nl} [{}^0T]^T \cdot [{}^uT]^T \cdot [{}^n_n\bar{B}_L(0)]^T \cdot \{{}^n_n\bar{R}(0)\} dl \tag{2}$$

where the value in parenthesis in the displacement-strain matrix $[{}^n_n\bar{B}_L]$ indicates the location of the integration point, and that in the resultant force vector $\{{}^n_n\bar{R}\}$ indicates the point at which stresses are evaluated, respectively. $[{}^0T]$ and $[{}^uT]$ are the transformation matrices based upon the updated Lagrangian formulation.

Using elementary beam theory, relations between bending moments $\bar{R}_1$, $\bar{R}_2$ and shear forces $\bar{R}_5$, $\bar{R}_6$ can be expressed as

$$\bar{R}_5 = -\frac{d\bar{R}_2}{dz}, \quad \bar{R}_6 = -\frac{d\bar{R}_1}{dz} \tag{3}$$

Thus, the distributions of bending moment increments $\Delta_n\bar{R}_1(s)$ and $\Delta_n\bar{R}_2(s)$ along the element length can be approximated by the following equations using the bending moment increments $\Delta_n\bar{R}_1(0)$, $\Delta_n\bar{R}_2(0)$ and the shear force increments $\Delta_n\bar{R}_5(0)$, $\Delta_n\bar{R}_6(0)$ at the midpoint of the element:

$$\Delta_n\bar{R}_1(s) = \Delta_n\bar{R}_1(0) - \frac{\Delta_n\bar{R}_6(0)^n ls}{2} \tag{4a}$$

$$\Delta_n\bar{R}_2(s) = \Delta_n\bar{R}_2(0) - \frac{\Delta_n\bar{R}_5(0)^n ls}{2} \tag{4b}$$

where ${}^n l$ is element length at $t=t_n$. Equations (4) show that bending moments are subject to a linear change in an element and are likely to take the maximum value on either ends $(s=\pm 1)$. As other resultant forces have constant values in the element, a fully plastic state can be determined with a yield function by comparing the calculated distributions from Eqs.(4).

In dynamic collapse analyses using the ASI technique, an explosion or a fracture is expressed by shifting the numerical integration point according to Eq.(1) immediately after the occurrence of a fractured section on either end of the element, and reducing the resultant forces of the element simultaneously. For instance, if a fully plastic section or a fractured section has first occurred at the left end of an element (r=-1), the numerical integration point is shifted immediately to the right end of the element (s=1) according to Eq.(1). The released force vector which operates on the element at the next step in the analysis is then expressed by the following equation:

$$\{{}^n_nF\} = \int_{{}^n l} [{}^0T]^T \cdot [{}^uT]^T \cdot [{}^n_n\bar{B}_L(1)]^T \cdot \{{}^n_n\bar{R}(-1)\} dl \tag{5}$$

Similarly, if a fully plastic section or a fractured section has first occurred at the right end of the element (r=1), the numerical integration point is shifted to the left end of the element (s=-1).

In case of the implicit scheme, the incremental stiffness matrices used in the algorithm, for the case when the entire region in an element is elastic, are

$$[{}^n_n\bar{K}_L] = \int_{{}^n l} [{}^uT]^T \cdot [{}^0T]^T [{}^n_n\bar{B}_L(0)]^t [D^e(0)][{}^n_n\bar{B}_L(0)][{}^0T] \cdot [{}^uT] dl \tag{6a}$$

$$[{}^n_n\bar{K}_{NL}] = \int_{{}^n l} [{}^uT]^T \cdot [{}^0T]^T [{}^n_n\bar{G}(0)]^t [{}^n_n\bar{S}(0)][{}^n_n\bar{G}(0)][{}^0T] \cdot [{}^uT] dl \tag{6b}$$

where $[D^e]$, $[{}^n_n\bar{G}]$, $[{}^n_n\bar{S}]$ are the elastic stress-strain, initial displacement and initial stress matrices, respectively. In case the section is fully plastic or has fractured first at the left end of the element (r=-1), the incremental stiffness matrices are given by the following equation:

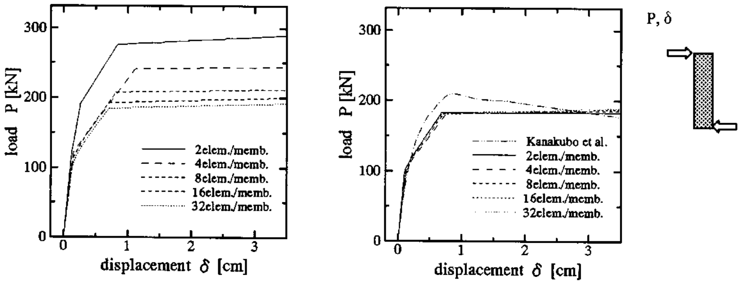
$$[{}^n_n\bar{K}_L] = \int_{{}^nl} [{}^uT]^T \cdot [{}^0T]^T [{}^n_n\bar{B}_L(1)]^t [D^p(-1)] [{}^n_n\bar{B}_L(1)] [{}^0T] \cdot [{}^uT] dl \tag{7a}$$

$$[{}^n_n\bar{K}_{NL}] = \int_{{}^nl} [{}^uT]^T \cdot [{}^0T]^T [{}^n_n\bar{G}(1)]^t [{}^n_n\bar{S}(-1)] [{}^n_n\bar{G}(1)] [{}^0T] \cdot [{}^uT] dl \tag{7b}$$

where $[D^p]$ is the plastic stress-strain matrix. It should be noted that when new hinges are formed the resultant force increments calculated at the new integration point are automatically added to those originally existing at the very point. As a result of using this procedure, a "non-smoothness" type of phenomenon does not appear in the calculation. More details of the implicit ASI algorithm are explained in the author's previous papers (Toi and Isobe 1993, Isobe and Toi 2000).

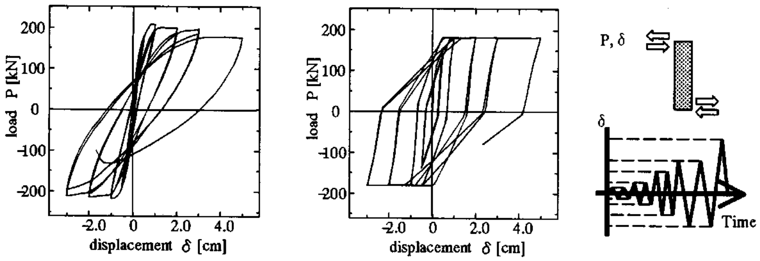
STATIC AND QUASI-STATIC ANALYSES OF RC BEAMS

A tri-linear type model which has a crack point and an yield point is employed to RC beams subjected under non-cyclic loads. Figure 2 shows the load-displacement curves obtained by the conventional code and by the ASI technique, in case of a simply supported column subjected to shear force. The results show that the converged solution can be obtained by only



(a)Conventional code (b) ASI technique

Fig.2 Simply supported column subjected to shear force



(a) Experiment (Kanakubo *et al.*) (b) ASI technique (2elem./memb.)

Fig.3 RC column under repeated quasi-static load two-element subdivision per member in the ASI technique, while more elements are required in order to obtain the converged solution in the conventional finite element code. The validity of the tri-linear type model used in the analysis is confirmed by comparing the result with that of the experiment (Kanakubo and Sonebe 1992).

A degrading tri-linear model is applied to RC beams subjected under cyclic loads. Figure 3 shows load-displacement curves obtained from the experiment of an RC column under repeated quasi-static load (Kanakubo and Sonebe 1992), along with a numerical result obtained by the ASI technique with two-element subdivision per member. The validity of the degrading tri-linear model can be verified by the comparison.

SEISMIC COLLAPSE ANALYSIS CONSIDERING MEMBER FRACTURE AND CONTACT

Phenomena with strong nonlinearity and discontinuities such as member fracture are easily analyzable using the proposed code. However, behavior such as the penetration of a member through a floor could be observed in the analyses (Isobe and Toi 2000), since contact between members was not considered. Actually, some structures observed in the Great Hanshin-Awaji Earthquake were collapsed in the intermediate layer and as a result, the upper layer piled up on the layer. Although there are some numerical examples using the Distinct Element Method (DEM) that consider contact between members, there are no application examples using the FEM that can continuously analyze the behavior from the elastic stage to the collapse stage. Thus, a contact algorithm is added to the proposed code and applied to the seismic damage analysis to reproduce the intermediate-layer failure phenomena.

Member fracture is assumed to occur at certain occasions when at least one of the following equations has been satisfied.

$$\left(\frac{df_x}{df_{xexp}}\right) - 1 \geq 0, \quad \left(\frac{df_y}{df_{yexp}}\right) - 1 \geq 0 \tag{8a}$$

$$\left(\frac{\gamma_x}{\gamma_{xexp}}\right) - 1 \geq 0, \quad \left(\frac{\gamma_y}{\gamma_{yexp}}\right) - 1 \geq 0 \tag{8b}$$

where df_{xexp} and df_{yexp} are the critical ductility factors for x–and y-axes, γ_{xexp} and γ_{yexp} are the critical shearing strain for x– and y-axes, respectively. Values of 5.0(column) and 20.0(beam) for the critical ductility factors, and 4.0×10^{-3} for the critical shearing strain are used in this paper.

When member fracture occurs according to the conditions given above, it is expressed by shifting the numerical integration point of the fractured element with a simultaneous release of the resultant forces in the element, as shown in Figure 4. The distribution of element mass at each node point is controlled to split into half of the mass subjected at the node point before the fracture. It is also to be noted that as elements and node points are still treated as a continuous model in the calculation process, new virtual node points for the fractured sections are needed to be established at the post-processing stage. The elements with the virtual node points are then visualized as rigid bars thereafter.

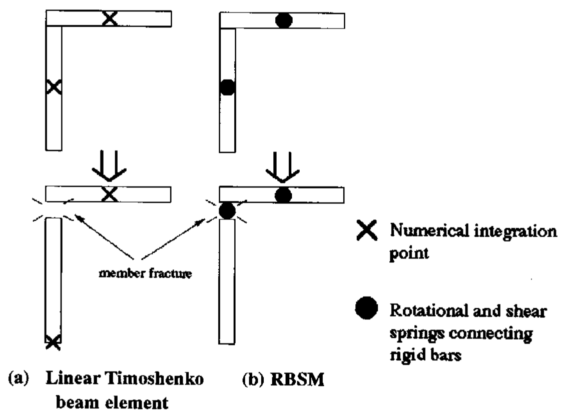


Fig.4 ASI technique dealing with member fracture

Contact of fractured elements is judged by using post-processing data, which includes virtual node points for the fractured elements, and thus expressing the actual fractured phenomena. Figure 5 shows the relation of coordinates between fractured element and other elements. A_1 and A_2 denote both node points of a fractured element E_f. Likewise, B_{i1} and B_{i2} denote both node points of other elements E_i(i=1,2,3,$\cdots$n). Using the coordinates of each node point; $A_1(x_{f1},y_{f1},z_{f1})$, $A_2(x_{f2},y_{f2},z_{f2})$, $B_{i1}(x_{i1},y_{i1},z_{i1})$, $B_{i2}(x_{i2},y_{i2},z_{i2})$, the condition of four node points of both elements existing on a same plane can be defined as the equation below:

$$
\begin{aligned}
f(x,y,z) \equiv \\
&\{(y_{i1}-y_{f2})(z_{i2}-z_{f2})-(y_{i2}-y_{f2})(z_{i1}-z_{f2})\}(x_{f1}-x_{f2}) \\
&+\{(x_{i2}-x_{f2})(z_{i1}-z_{f2})-(x_{i1}-x_{f2})(z_{i2}-z_{f2})\}(y_{f1}-y_{f2}) \\
&+\{(x_{i1}-x_{f2})(y_{i2}-y_{f2})-(x_{i2}-x_{f2})(y_{i1}-y_{f2})\}(z_{f1}-z_{f2}) = 0 \qquad (9)
\end{aligned}
$$

Also, the condition of elements existing in a specific distance from a fractured element, is assumed as the equation below:

$$|\overline{A_1B_{i1}}| + |\overline{A_1B_{i2}}| + |\overline{A_2B_{i1}}| + |\overline{A_2B_{i2}}| \leq C_l(L_f + L_i) \qquad (10)$$

where $|\overline{A_1B_{i1}}|$, $|\overline{A_1B_{i2}}|$, $|\overline{A_2B_{i1}}|$, $|\overline{A_2B_{i2}}|$ are the distance between each node point, and L_f, L_i(i=1,2,3,$\cdots$n) are the length of fractured element E_f and other elements E_i(i=1,2,3$\cdots$n), respectively. C_l is a coefficient related to contact length, and the value of 1.8 is used.

On the other hand, another condition is given to the elements, which exist on a same plane from the initial stage of the analysis. The condition of elements existing in a specific distance from a fractured element on a same plane, is given by:

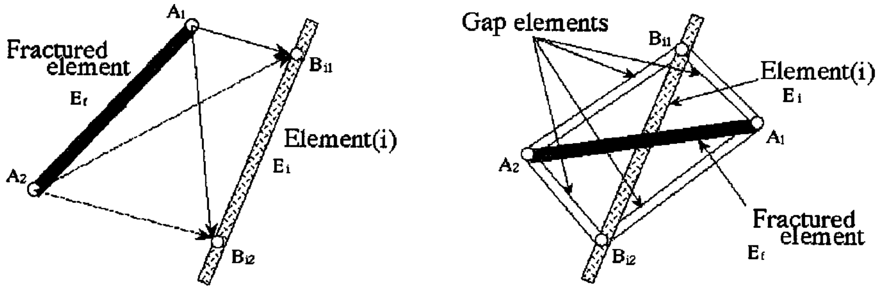


Fig.5 Fractured element and gap elements in contact algorithm

$$|\overline{A_1 B_{i1}}| + |\overline{A_1 B_{i2}}| \leq L_i \tag{11a}$$

$$|\overline{A_2 B_{i1}}| + |\overline{A_2 B_{i2}}| \leq L_i \tag{11b}$$

In this algorithm, contact loop is automatically avoided if the distance between node points is not decreasing. For the elements that exist on a same plane from the initial stage (Eq.(9)), the elements are judged to be in contact if Eq.(11) is satisfied. For other elements which does not exist on a same plane, and if only the distance between node points satisfies Eq.(10), the elements are judged to be in contact by the next condition, which refers to the four node points nearly forming a plane:

$$f(x, y, z) \leq C_f \tag{12}$$

C_f is a coefficient expressing the degree of the planar shape, which is fixed by considering the sectional properties of constituting members. Values of $2.0 \times 10^2 \, m^3$ for columns and $5.0 \times 10^4 \, m^3$ for beams are used in this paper.

Consequently, the contact between members is judged by two sets of conditions; Eqs.(9), (11) and Eqs.(10), (12). Once two elements are judged to be in contact, a total of four gap elements are fixed between the node points, as shown in Figure 5. Material properties for the gap elements are assumed as the same properties with other elements.

Seismic damage analyses are carried out to an eight-story three-span RC building subjected under El Centro seismic wave. The seismic wave is given precisely at the fixed points on the ground floor. The total number of elements is 464, and that of node points is 340. Figure 6(a) shows a case in which only member fracture is considered, and Figure 6(b) shows the result when the contact algorithm is added to the code. By considering the contact between members, the reproduction of seismic damage observed in actual earthquakes such as intermediate-layer failure became possible. The computing time using SUN ultra 5 (CPU: 270MHz, memory: 128MB) was approximately 50 minutes.

CONCLUSION

In this paper, a nonlinear finite element code using the ASI technique with RC beam model is developed, in order to analyze seismic damage problems including structural

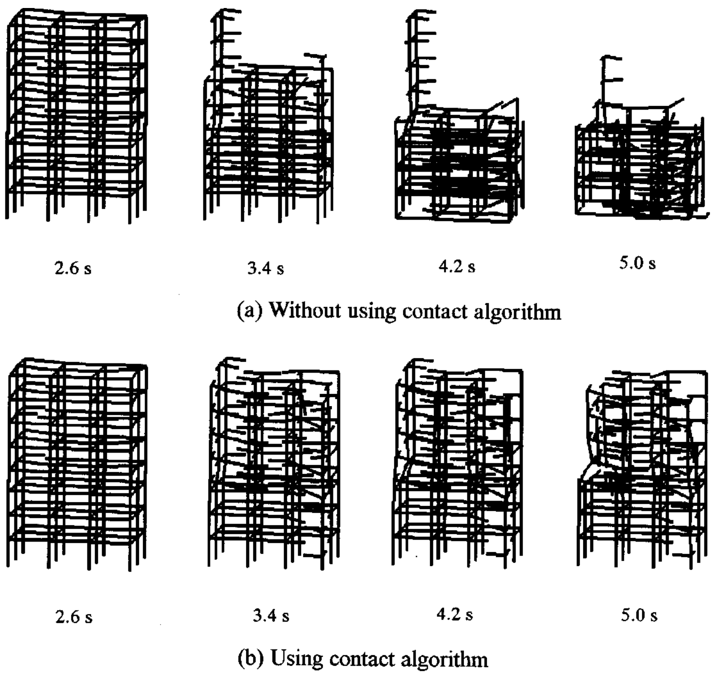


Fig.6 Seismic damage analysis of RC framed structure

discontinuities. The fracture of a section is modeled by shifting the numerical integration point with a simultaneous release of the resultant forces. The proposed code is improved by considering the contact between members in order to obtain results that agree more closely with actual behavior. The results reveal that this technique can be used in the numerical estimation of structural reliabilities.

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