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A Finite Element Scheme for Calculating Inverse Dynamics of Link Mechanisms

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Abstract
This paper describes a three-dimensional parallel solution scheme for inverse dynamics of link mechanisms, which has already been proposed for the two-dimensional case and applied in several in-plane motions. In this theory, the entire system is subdivided into finite elements and evaluated as a continuum. A single-link structure of a pin joint and a rigid bar is expressed using the Shifted Integration (SI) technique, which is conventionally used in finite element analyses of framed structures. This scheme calculates nodal forces by evaluating equations of motion in a matrix form, and thus information from the entire system can be handled in parallel, which is a very useful characteristic when applied in closed-loop or continuously transforming mechanisms. The obtained nodal forces are then converted into the joint torque in the system. Simple numerical tests on two-dimensional and three-dimensional open-loop link mechanisms are carried out for comparison with other schemes. The proposed scheme is implemented in a control system to evaluate the performance in actual control with dynamics compensation, and some control experiments are carried out on an open-loop link mechanism. The results reveal the possibility of using the proposed solution scheme in feed-forward control, independent of the system configuration of link mechanisms.

1 Introduction

There is a difficulty in calculating the inverse dynamics for the closed-loop mechanism using conventional methods such as the Newton-Euler method or the Lagrangian method. This is due to the presence of interdependent variables between the constituting links, which become impossible to derive by the former method when a chain is closed in the system. The latter method is also difficult to apply, since the derivation process of the equation considering the binding condition is very complicated. Generally, robotic tasks include motions that generate open and closed loops alternately, and the dynamic equations of the system (or the numerical algorithm) require instant revision during the motion. A unified solution scheme for calculating the inverse dynamics is strongly desired, particularly for those cases of massive, quick-motion robots controlled by force.

Isobe and Nakagawa proposed the application of the Finite Element Method (FEM), a widely used computational tool for analyzing, for example, structures and fluids, to a control system of connected piezoelectric actuators, and achieved good control not only of the actuator itself but also of the entire system [1]. Isobe *et al.* implemented the FEM in a two-dimensional solution scheme of inverse dynamics for hyper-redundant link mechanisms [2], and also developed it for application to in-plane problems of closed-loop link mechanisms [3]. Using the characteristic of the FEM, i.e., the capability of expressing the behavior of each discrete element as well as that of the entire continuous system, local information such as nodal forces or displacements can be calculated in parallel. The nodal forces are calculated incrementally in a matrix form, which does not require any revision of the outside frame, and the variables inside can be revised simply by changing the input data in the case of a physical change in the hardware system. The obtained nodal forces are then used to calculate the joint torque in the link systems.

In this paper, we describe a three-dimensional solution scheme using the FEM for inverse dynamics of link mechanisms. Link mechanisms are modeled using linear Timoshenko beam elements based on the Shifted Integration (SI) technique [4], which is conventionally used in finite element analyses of framed structures. Nodal forces for obtaining target trajectories are calculated using the FEM, and the joint torque of each link is calculated based on a matrix-formed conversion equation between nodal forces and the joint torque. Some numerical tests are carried out for several types of open-loop link mechanisms to verify the validity and flexibility of the scheme. The proposed scheme is also implemented in a control system to evaluate the performance in actual control with dynamics compensation, and some control experiments are carried out on a two-dimensional, nongear open-loop link mechanism.

2 Finite Element Modeling of Link Mechanisms

The SI technique, which is applied in order to model link mechanisms in this study, was originally developed as a finite element scheme for the analysis of framed structures. By considering the equivalence conditions between the strain energy approximations of a linear Timoshenko beam element and a physical model, the rigid-bodies spring model (RBSM), the relationship between the locations of a numerical integration point (s_1) and a plastic hinge (r_1) in the linear Timoshenko beam element $(-1 \leq r_1, s_1 \leq 1)$ is obtained [4]. Referring to Fig. 1, it is expressed by

$$s_1 = -r_1 \ or \ r_1 = -s_1, \tag{1}$$

where s_1 and r_1 are the positions of the numerical integration point in the finite element and the spring in the RBSM, respectively. Referring to the above equation, the rotational and shear spring placed at

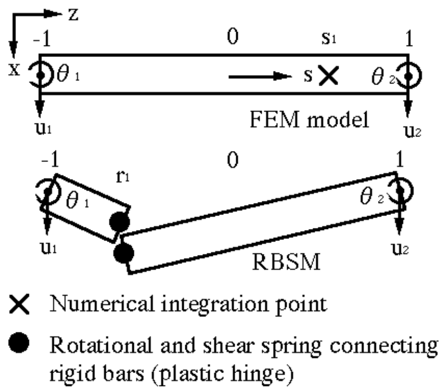


Figure 1: Linear Timoshenko beam element and its physical equivalent.

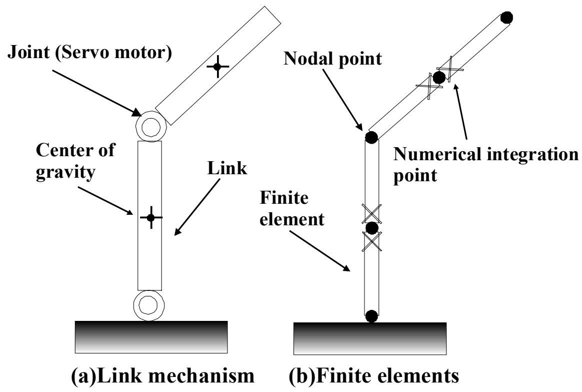


Figure 2: Modeling of link mechanism by Shifted Integration technique.

the left end (r_1= -1) of the element can be expressed by shifting a numerical integration point in the element to the right end (s_1=1). Various stiffness values of a link joint are then expressed by changing the stiffness of the spring (or the element). Figure 2 shows the general concept of modeling by the SI technique. As shown in the figure, a link mechanism formed by a motor and a link member can be modeled by placing a nodal point at the center of gravity, and by two Timoshenko beam elements with numerical integration points shifted to the opposite ends of the link joint. The elemental stiffness matrix is obtained using s_1 , r_1 and the normalized stiffness C_{mot} of the spring:

$$[K] = C_{mot} \int_V [B(s_1)]^T [D(r_1)][B(s_1)] dV \quad . \tag{2}$$

Various types of link joints (pin to rigid) can be expressed by varying C_{mot} between 0 and 1. The value 0 is used in this study to estimate the validity of the proposed scheme in the pin joint-rigid bar link mechanisms. A lumped mass matrix is also defined using the location of the numerical integration point s_1. The diagonal components of the elemental mass matrix are

$$[M] = [m_1 \; m_1 \; m_1 \; \frac{m_1 l^2}{12} \; \frac{m_1 l^2}{12} \; t_1 \; m_2 \; m_2 \; m_2 \; \frac{m_2 l^2}{12} \; \frac{m_2 l^2}{12} \; t_2] , \tag{3}$$

where

$$m_1 = \rho Al(1-s_1)/2, \quad m_2 = \rho Al(1+s_1)/2,$$
$$t_1 = \rho I_z l(1-s_1)/2, \quad t_2 = \rho I_z l(1+s_1)/2, \tag{4}$$

and ρ, A, l, and I_z are the density of the member, the cross-sectional area, the length of the element and the polar moment of area inertia, respectively. Based on the matrix, the total mass of the element becomes concentrated at r_1=1 when the link joint is placed at r_1= -1 (thus s_1=1). A nodal point placed between two Timoshenko beam elements thus expresses the center of gravity in a link member (see Fig. 2). Additional mass representing the motors or other objects can be applied at corresponding nodal points.

3 Three-Dimensional Parallel Solution Scheme for N-Link Mechanism

Figure 3 shows the nodal forces (based on global coordinates) acting on the i-th link (i=1~n) in a three-dimensional open-loop n-link mechanism. The joint torque τ_{ix} required around the x-elemental axis on the i-th link, for example, is determined by adding an i+1-th joint torque $\tau_{(i+1)x}$ to the sum of inertia moments acting on this link, and is expressed by nodal forces F_{iy} and $F_{i\Phi x}$ based on elemental (or link) coordinates as follows:

$$\tau_{ix} = l_{iC}F_{iy} + l_i(\sum_{j=i+1}^{n} F_j)_y + F_{i\phi x} + \tau_{(i+1)x} \ , \tag{5}$$

where l_{iC} is the length between the former joint and the center of gravity and l_i is the link length. By

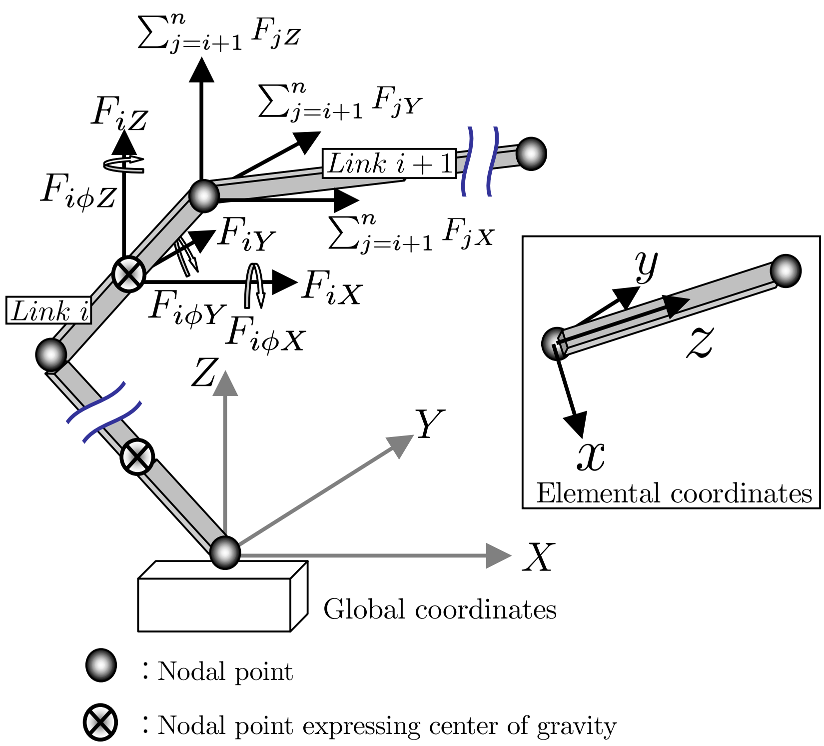


Figure 3: Nodal forces acting on i -th link in an open-loop n-link mechanism.

considering other components around the y- and z-axes, and arranging them into global coordinates (X, Y, Z) in a matrix form, the joint torque vector is expressed as

$$\{\tau^n\} = [L^n][T^n]\{P^n\}, \tag{6}$$

where $\{P^n\}$ is a vector related to nodal force, defined as

$$\{P^n\} = \left\{ \begin{array}{c} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_n \end{array} \right\}, \quad where \quad \{P_i\} = \left\{ \begin{array}{c} F_{iX} \\ F_{iY} \\ F_{iZ} \\ \sum_{j=i+1}^{n} F_{jX} \\ \sum_{j=i+1}^{n} F_{jY} \\ \sum_{j=i+1}^{n} F_{jZ} \\ F_{i\phi X} \\ F_{i\phi Y} \\ F_{i\phi Z} \end{array} \right\}. \tag{7}$$

The transformation matrix $[T^n]$ is expressed as

$$[T^n] = [h^n][T^n_{GE}], \tag{8}$$

where $[h^n]$ is a correction matrix between x-y and z-x coordinate systems, which simply inverts their signs in the y-axis direction. $[T^n{}_{GE}]$ is a transformation matrix between global and elemental coordinates which is expressed as

$$\left[\ T^n_{GE}\ \right] = \left[\begin{array}{ccccccc} T_1 & & & & & & \\ & T_2 & & & 0 & & \\ & & T_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & 0 & & & & \cdot & \\ & & & & & & T_n \end{array} \right], \tag{9}$$

where

$$\left[\ T_i\ \right] = \left[\begin{array}{ccc} A_i & 0 & 0 \\ 0 & A_i & 0 \\ 0 & 0 & A_i \end{array} \right], \quad and \quad \left[\ A_i\ \right] = \left[\begin{array}{ccc} cos\phi_{iXx} & cos\phi_{iYx} & cos\phi_{iZx} \\ cos\phi_{iXy} & cos\phi_{iYy} & cos\phi_{iZy} \\ cos\phi_{iXz} & cos\phi_{iYz} & cos\phi_{iZz} \end{array} \right], \tag{10}$$

where ϕ_{iXx}, for example, represents the rotational angle between X-global and x-elemental coordinates. $[L^n]$ is a matrix related to member length and is expressed as

$$[L^n] = [T^n_\Lambda][\Lambda^n], \tag{11}$$

where $[T^n_\Lambda]$ is a transformation matrix between each elemental coordinate, and is expressed as

$$\left[\begin{array}{c} T^n_\Lambda \end{array}\right] = \left[\begin{array}{cccccc} T_{11} & T_{12} & T_{13} & \cdot & \cdot\ \cdot & T_{1n} \\ & T_{22} & T_{23} & \cdot & \cdot\ \cdot & T_{2n} \\ & & T_{33} & \cdot & \cdot\ \cdot & T_{3n} \\ & & & \cdot & \cdot\ \cdot & \cdot \\ & & & & \cdot\ \cdot & \cdot \\ & 0 & & & \cdot & \cdot \\ & & & & & T_{nn} \end{array}\right]. \tag{12}$$

$[T_{ij}]$ (i,j=1~n) is expressed using matrix $[A_i]$ shown above:

$$[T_{ij}] = [A_i][A_j]^T. \tag{13}$$

$[\Lambda^n]$ is expressed as

$$\left[\begin{array}{c} \Lambda^n \end{array}\right] = \left[\begin{array}{ccccccc} \Lambda_1 & & & & & & \\ & \Lambda_2 & & & 0 & & \\ & & \Lambda_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & 0 & & & & \cdot & \\ & & & & & & \Lambda_n \end{array}\right], \tag{14}$$

where

$$\left[\begin{array}{c} \Lambda_i \end{array}\right] = \left[\begin{array}{ccccccccc} 0 & l_{iC} & 0 & 0 & l_i & 0 & 1 & 0 & 0 \\ l_{iC} & 0 & 0 & l_i & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]. \tag{15}$$

Information on the i+1~n link is summed by multiplying the $[L^n]$ matrix with vector $[T^n]\{P^n\}$, which is the nodal force vector transformed into elemental coordinates. In cases of closed-loop link mechanisms, the above matrix is divided into multiple parts, as shown below, to fix the configuration of passive joints as well as the torque allocation undertaken by active joints.

$$[L^n] = \left[\begin{array}{cc} L^a & 0 \\ 0 & L^b \end{array}\right] \tag{16}$$

The suffixes a and b are the number of links (a+b=n) when the mechanism is divided into two parts. This process is the only difference in the algorithm between open- and closed-loop link mechanisms. A vector related to incremental nodal forces acting on the i-th link is defined using the nodal numbers k (=2i):

$$\{\Delta p_k\} = \left\{ \begin{array}{c} \Delta f_{kX} \\ \Delta f_{kY} \\ \Delta f_{kZ} \\ \sum_{h=k+1}^{2n+1} \Delta f_{hX} \\ \sum_{h=k+1}^{2n+1} \Delta f_{hY} \\ \sum_{h=k+1}^{2n+1} \Delta f_{hZ} \\ \Delta f_{k\phi X} \\ \Delta f_{k\phi Y} \\ \Delta f_{k\phi Z} \end{array} \right\}. \tag{17}$$

Thus, the vector related to the nodal force acting on the *i*-th link at t+Δt is successively calculated using the above vector as follows:

$$\{P_i\}_{t+\Delta t} = \{P_i\}_t + \{\Delta p_k\}. \tag{18}$$

The successive values of the n-link joint torque are then obtained by substituting Eq. (18) into Eq. (6). Newmark's β method (β=1/4) is used as the time integration scheme to solve the incremental kinematic equation.

4 Numerical Examples

The parallel solution scheme is implemented into a computational program, and applied to the joint torque calculation of a two-dimensional open-loop mechanism as an example, in order to confirm the accuracy of the calculated torque curves by comparing them with those obtained by the Newton-Euler method. Figures 4(a) and 4(b) show the torque curves obtained using the two schemes for the target trajectory for the 1.0 s motion given in the eight-link mechanism (length of each link: 20cm; weight: 107.5g; center of gravity at midpoint), as shown in Fig. 4(a). Gravity is assumed to act vertically downward. Although the motion may produce various nonlinear forces, the torque curves obtained by the FEM are in good agreement with those obtained by the Newton-Euler method. Evidently, the parallel solution scheme is capable of considering every component in the dynamics.

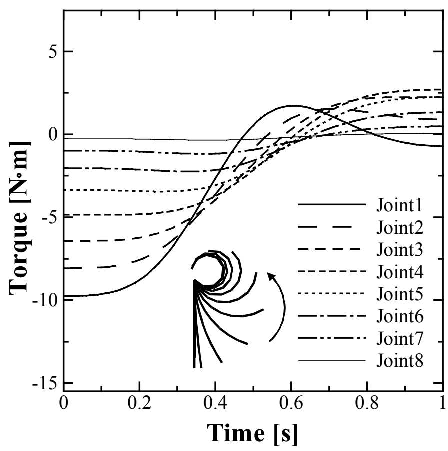


Figure 4(a): Joint torque curves obtained by Newton-Euler method.

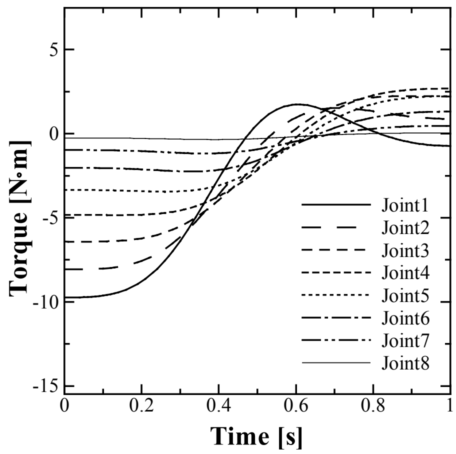
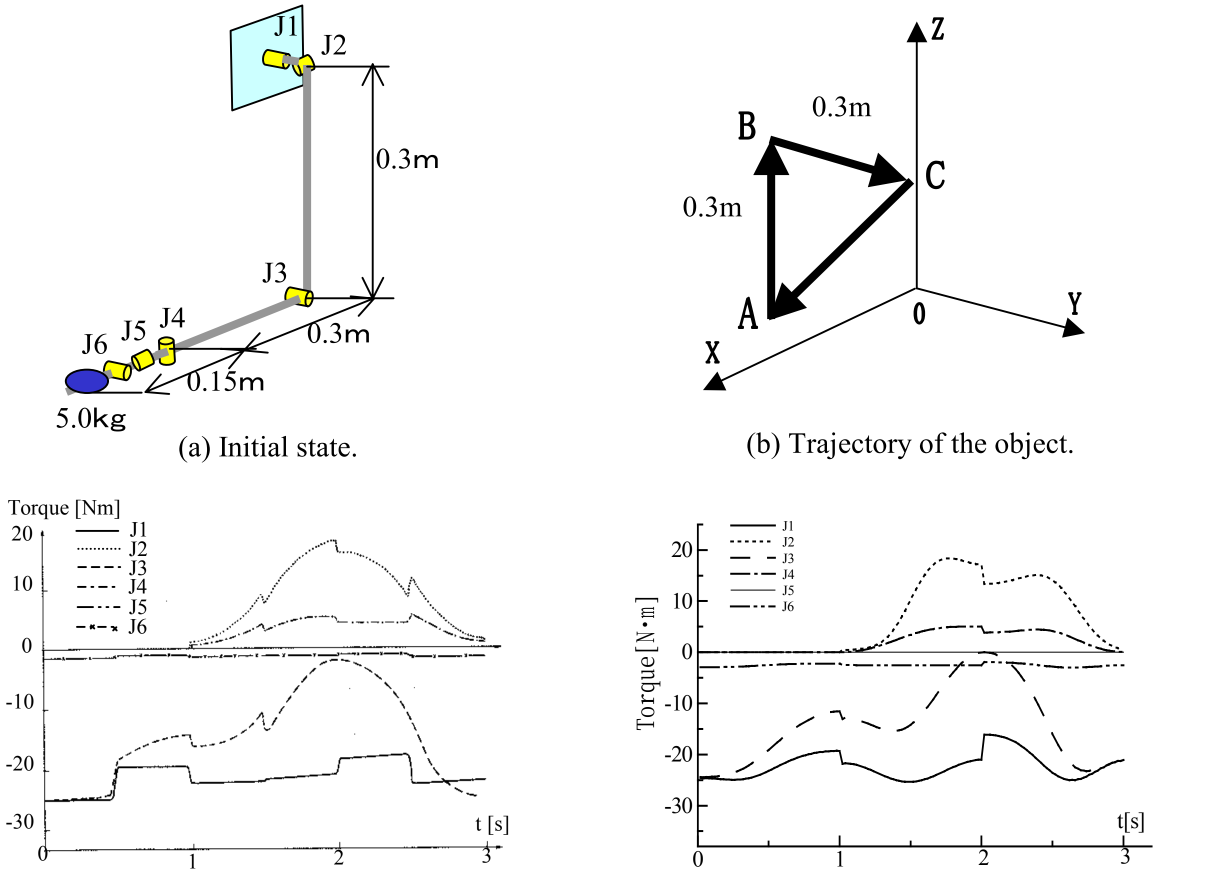


Figure 4(b): Joint torque curves obtained by FEM.



(a) Initial state.

(b) Trajectory of the object.

(c) Joint torque curves [5].

(d) Joint torque curves obtained by FEM.

Figure 5: Calculation of joint torque curves for a 6-DOF robotic arm.

Figures 5(a) to 5(d) show the numerical results obtained by applying a three-dimensional motion to a 6-DOF robotic arm. The trajectory of an object (mass: 5.0kg) which is held at the end of the arm is approximately given as shown in Fig. 5(b). Although the actual trajectory is not precisely along a straight line as shown, the torque curves calculated by the FEM show good agreement with those in the reference [5]. It is confirmed that the proposed parallel solution scheme is also valid for three-dimensional cases.

5 Control Experiments

Conventional schemes for calculating inverse dynamics supply exact solutions through the use of the rotational angle, the angular velocity and the angular acceleration at a specific time as input. In the meantime, the proposed parallel solution scheme yields an approximate solution by summing the incremental information in each step. Therefore, the slight difference in the calculation procedure must be investigated to enable application in a control system.
The control system shown in Fig. 6 is developed to determine the performance of the system in actual control experiments. A three-link mechanism with no gear shaft attached to the motors is used in the experiments to maximize the effect of the dynamics.
Input torque τ is calculated using

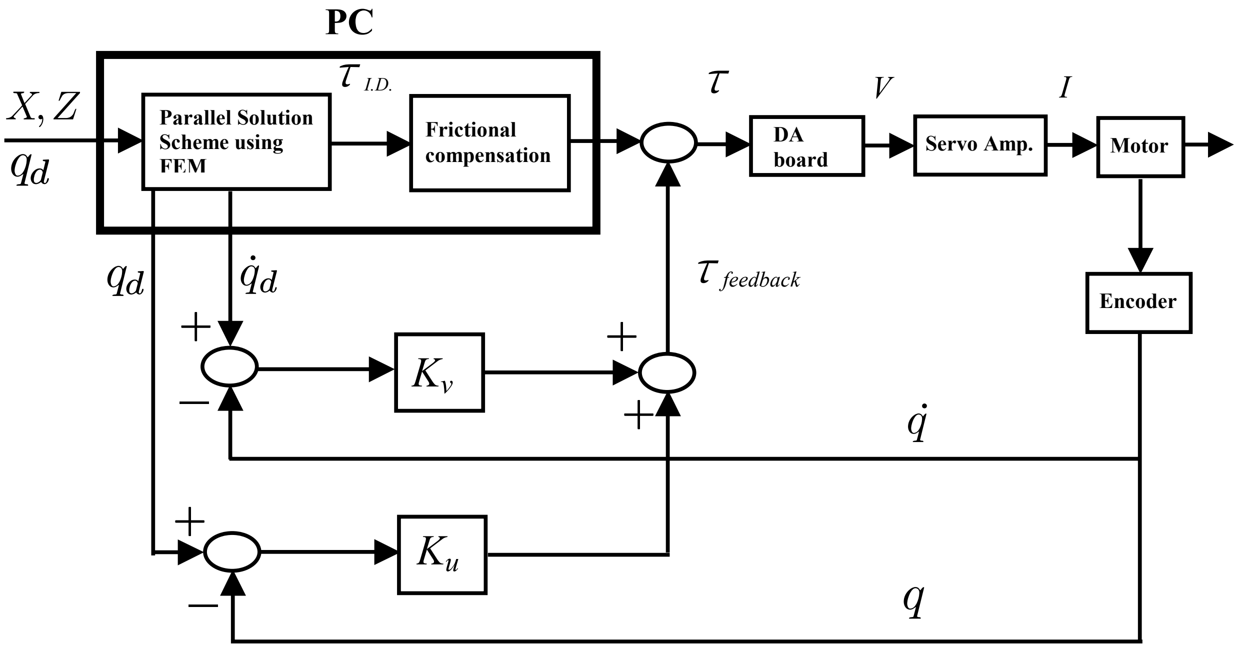


Figure 6: Outline of the control system with parallel solution scheme.

$$\tau = J\ddot{q}_d + D\dot{q}_d + f_c + \tau_{I.D.} + \tau_{feedback}, \tag{19}$$

where J is the inertial moment of motors, D is the viscosity coefficient of motors, f_c is the dynamic friction force, $\tau_{I.D.}$ is the input torque calculated by FEM, and q_d and $\dot{q}_d$ are the target angle and angular velocity, respectively. The viscosity coefficient of motors D and the dynamic friction force f_c are identified beforehand by simple experiments. $\tau_{feedback}$ is the feedback torque which is obtained using

$$\tau_{feedback} = K_u(q_d - q) + K_v(\dot{q}_d - \dot{q}), \tag{20}$$

where q and $\dot{q}$ are the actual angle and angular velocity acquired from the attached encoders,

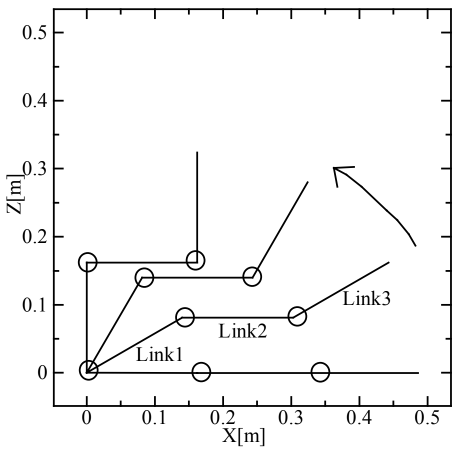


Figure 7: 3.0 s target motion for a 3-link mechanism.

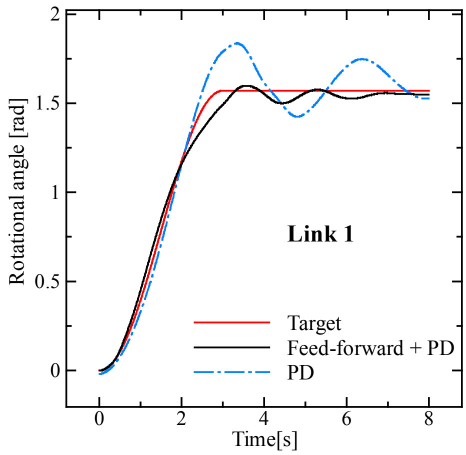


Figure 8: Control result for Link 1.

respectively. K_u and K_v are the feedback gain for the angle and angular velocity, respectively.
A 3.0 s target motion is applied to the 3-link mechanism, as shown in Fig. 7. PD control with feed-forward using calculated inverse dynamics is compared to control without feed-forward. Figure 8 shows the control result for Link 1 as an example. It is evident that (feed-forward + PD) control gives much better convergence against the target motion.
Although the proposed scheme requires incremental and successive calculation in the algorithm, the result of the control experiment clearly shows that the performance of a control system with this scheme presents no problem in actual use. The calculation time per step is about 0.5 ms, which is short enough to satisfy the sampling time in the system (which is 10 ms).

6 Concluding Remarks

The proposed solution scheme derives nodal forces in parallel and converts them to the joint torque, which works conveniently in application to many types of link mechanisms. It requires no revision in the basic numerical algorithm during the transformation process of the mechanisms. It may achieve stability and smoothness in continuous motions of robotic architecture. The obtained control results indicate the possibility of using the proposed solution scheme in feed-forward control independent of the system configuration of link mechanisms. Further control experiments on closed-loop and combined-loop link mechanisms are scheduled.

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