The Twenty-Sixth KKHTCNN Symposium on Civil Engineering
November 18-20, 2013, Singapore

Damage influence on vibration indices of a passing vehicle

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ABSTRACT

The new vibration index, spatial singular mode, is proposed in this paper. The numerical simulation of VBI (Vehicle-Bridge Interaction) system is also carried out and shows the higher sensitivity of the spatial singular mode to a local bridge damage than traditional vibration indices.

INTRODUCTION

There still exist technical issues in vibration-based SHM (Structure Health Monitoring) in accuracy and applicability to civil structures such as bridges. Although many traditional indices such as eigen-frequency are global, bridge damage often occurs in local. The spatial singular mode is proposed in this study to implement this settlement.

SPATIAL SINGULAR MODE

The spatial singular mode is a singular mode of corrected acceleration responses of the vehicle travelling through the bridge. Only two accelerometers and GPS are installed on the vehicle. Positions of vehicle axles are used to correct the vibration data.

Considering a half car model shown in **Fig. 1**, the equation of motion is described as

$$\mathbf{M}\ddot{\boldsymbol{z}}(\mathrm{t}) + \mathbf{C}\dot{\boldsymbol{z}}(t) + \mathbf{K}\boldsymbol{z}(t) = \boldsymbol{f}(t) \quad (1)$$

where $\boldsymbol{z}(t)$ and $\boldsymbol{f}(t)$ are the (displacement responses of the vehicle) and input (the external

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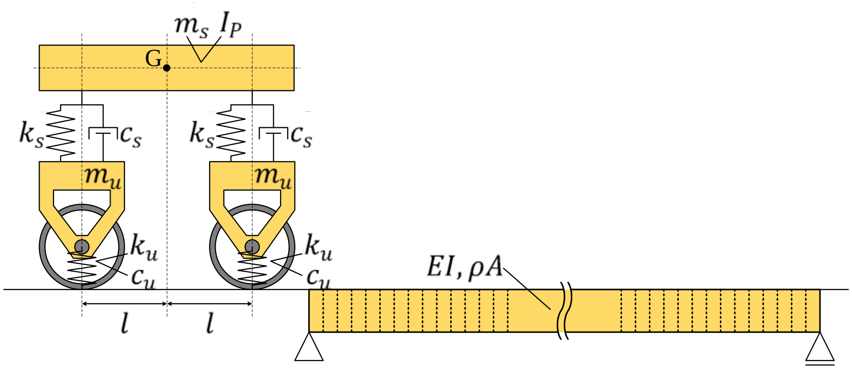


Fig. 1 VBI system

force) which are given by

$$\boldsymbol{z}(t) = \begin{Bmatrix} z_{s1}(t) \\ z_{u1}(t) \\ z_{s2}(t) \\ z_{u2}(t) \end{Bmatrix}, \boldsymbol{f}(t) = \begin{Bmatrix} 0 \\ c_u \dot{u}_1(t) + k_u u_1(t) \\ 0 \\ c_u \dot{u}_2(t) + k_u u_2(t) \end{Bmatrix} \tag{2}$$

where $z_{si}(t)$, $z_{ui}(t)$ and $u_i(t)$ are sprung-mass displacement, unsprung-mass displacement and input displacement, respectively. The subscript i denotes the front axle ($i = 1$) and the rear axle ($i = 2$). $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices and given by

$$\mathbf{M} = \begin{bmatrix} m_s/2 & 0 & m_s/2 & 0 \\ I_P/2l & 0 & -I_P/2l & 0 \\ 0 & m_u & 0 & 0 \\ 0 & 0 & 0 & m_u \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} c_s & -c_s & c_s & -c_s \\ lc_s & -lc_s & -lc_s & lc_s \\ -c_s & c_s + c_u & 0 & 0 \\ 0 & 0 & -c_s & c_s + c_u \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k_s & -k_s & k_s & -k_s \\ lk_s & -lk_s & -lk_s & lk_s \\ -k_s & k_s + k_u & 0 & 0 \\ 0 & 0 & -k_s & k_s + k_u \end{bmatrix} \tag{3}$$

where m_s, I_p, k_s, c_s, l, m_u, k_u and c_u are the sprung-mass, inertia, stiffness, damping, distance between the center of gravity and an axle, and the unsprung-mass, stiffness and damping, respectively.

Assuming one-dimensional simple beam as the bridge on the x-coordinate, the input displacement can be described as

$$u_i = R\big(x_i(t)\big) + y_{\mathrm{B}}(x_i(t), t) \tag{4}$$

where $R(x)$ and $y_{\mathrm{B}}(x, t)$ are the road unevenness and the bridge displacement. The vehicle pathway is also on the x-coordinate, and $x_i(t)$ denotes the axle position. The bridge response can be decomposed into modal responses such as

$$y_{\mathrm{B}}(x, t) = \sum_{k=1}^{\infty} \phi_k(x) q_k(t) \tag{5}$$

where $\phi_k(x)$ and $q_k(t)$ are k-th mode shape and basis coordinate of the bridge. Considering only first and second mode, the bridge component vector observed by the travelling vehicle is given by

Table 1 The standard parameters of vehicle

Sprung-	Mass	m_s	18,000[kg]
	Stiffness	k_s	1.0×10^6[kg/s²]
	Damping	c_s	1.0×10^4[kg/s]
	Inertia	I_P	64958[kg m²]
	Distance	l	1.875[m]
Unsprung-	Mass	m_u	1,100[kg]
	Stiffness	k_u	3.5×10^6[kg/s²]
	Damping	c_u	3.0×10^4[kg/s]
Run speed		v	10.0[m/s]

Table 2 The bridge parameters

Span Length	L	30.0[m]
Flexural Stiffness	EI	1.56×10^{10}[Nm]
Mass per unit length	ρA	3,000[kg/m]
Reyleigh coefficients	α	0.238
	β	0.000

$$\boldsymbol{y}_{\mathbf{B}}(t) = \boldsymbol{\Phi}(t)\boldsymbol{q}(t) = \begin{bmatrix} \phi_1(x_1(t)) & \phi_2(x_1(t)) \\ \phi_1(x_2(t)) & \phi_2(x_2(t)) \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \end{Bmatrix}. \tag{6}$$

The unsprung-response vector $\ddot{\boldsymbol{z}}_u(t)$ is

$$\ddot{\boldsymbol{z}}_u(t) = \begin{Bmatrix} \ddot{z}_{u1}(t) \\ z_{u2}(t) \end{Bmatrix} = \boldsymbol{\Phi}(t)\big(c_u\dot{\boldsymbol{q}}(t) + k_u\boldsymbol{q}(t)\big) + \boldsymbol{\epsilon}(t) \tag{7}$$

where $\boldsymbol{\epsilon}(t)$ is the error term including the sprung-mass response component and the road profile component. Shape function matrix is given by

$$\mathbf{N}(t) = \begin{bmatrix} -\frac{1}{L}\left(x_1(t) - \frac{2L}{3}\right) & \frac{1}{L}\left(x_1(t) - \frac{L}{3}\right) \\ -\frac{1}{L}\left(x_2(t) - \frac{2L}{3}\right) & \frac{1}{L}\left(x_2(t) - \frac{L}{3}\right) \end{bmatrix}. \tag{8}$$

where L is the span length of the bridge. Substitution of Eq. (8) into Eq. (7), we obtain

$$\ddot{\boldsymbol{z}}_{\boldsymbol{u}}(t) = \mathbf{N}(t)\mathbf{A}\big(c_u\dot{\boldsymbol{q}}(t) + k_u\boldsymbol{q}(t)\big) + \boldsymbol{\epsilon}(t). \tag{9}$$

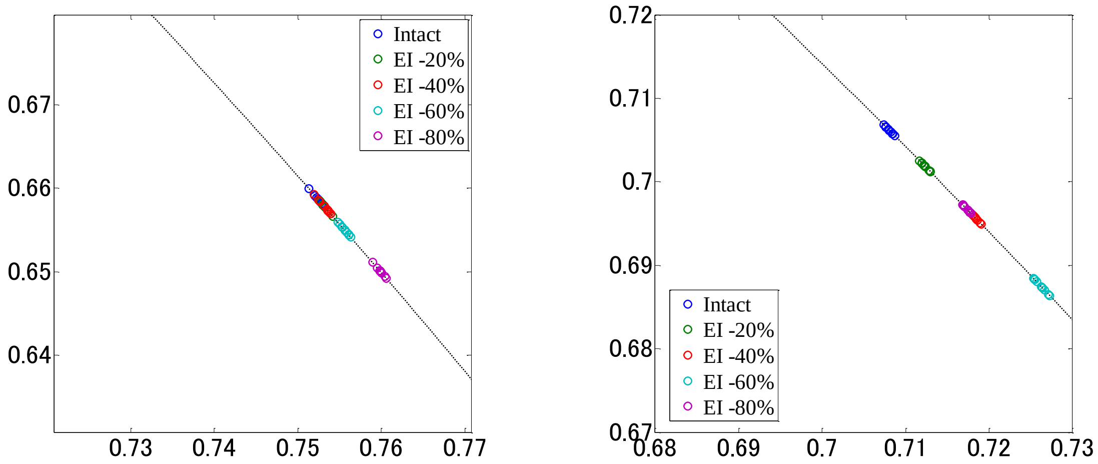
Assuming $\mathbf{N}^{-1}(t)\boldsymbol{\epsilon}(t) = \mathbf{A}\tilde{\boldsymbol{\epsilon}}(t)$ and $\boldsymbol{\sigma}(t) = c_u\dot{\boldsymbol{q}}(t) + k_u\boldsymbol{q}(t) + \tilde{\boldsymbol{\epsilon}}(t)$, Eq. (9) becomes

$$\mathbf{N}^{-1}(t)\ddot{\boldsymbol{z}}_u(t) = \mathbf{A}\boldsymbol{\sigma}(t). \tag{10}$$

$\mathbf{N}^{-1}(t)\ddot{\boldsymbol{z}}_u(t)$ is the corrected vehicle response vector. Applying SVD (Singular Value Decomposition) to $\mathbf{N}^{-1}(t)\ddot{\boldsymbol{z}}_u(t)$, $\mathbf{A}$ can be estimated as the spatial singular mode.

NUMERICAL SIMULATION

To examine the efficiency of spatial singular mode, the numerical simulation of VBI system is carried out. The parameters of the vehicle is shown in **Table 1**. The bridge is modeled by one-dimensional finite beam elements. The parameters of the bridge are also shown in **Table 2**. The first undamped natural frequency is 3.96[Hz]. The measurements are the acceleration



(a) Road profile: Extra Good (b) Road profile: Good

Fig. 4 The results of estimation for spatial singular mode (Damage Location: 3[m]=L/10)

responses of the unsprung-mass with 1% normal white noise. The bridge damage is introduced by decreasing EI locally.

RESULT AND DISCUSSION

Two mode shapes at two locations can be obtained from two accelerometers, shown in **Fig 3**. The mode shape in this study is normalized to satisfy $A_{1k}^2 + A_{2k}^2 = 1$. **Fig. 4** shows A_{11} and A_{21} relationships in different road profiles for each bridge state. The damage location is L/10 and the width is 1[m]. The mode is calculated 10 times for each case by Monte Carlo Simulation. The plot group for each bridge state are distinctively different. It means that the variety of the spatial singular mode is larger than the errors of measurements.

CONCLUSION

This study proposes applying spatial singular mode to bridge damage detection and examines the efficiency of the proposed method. According to the results, estimated spatial singular mode changes due to local damage larger than measurement errors. Thus, it is shown that high probability of the proposed method for bridge damage detection.

ACKNOLEDGMENT

This study is done by "Grant-in-Aid for Young Scientists(B)" (No. 25820200)

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