The Application of **the SVD-FDD Hybrid Method** to **Bridge Mode Shape Estimation**

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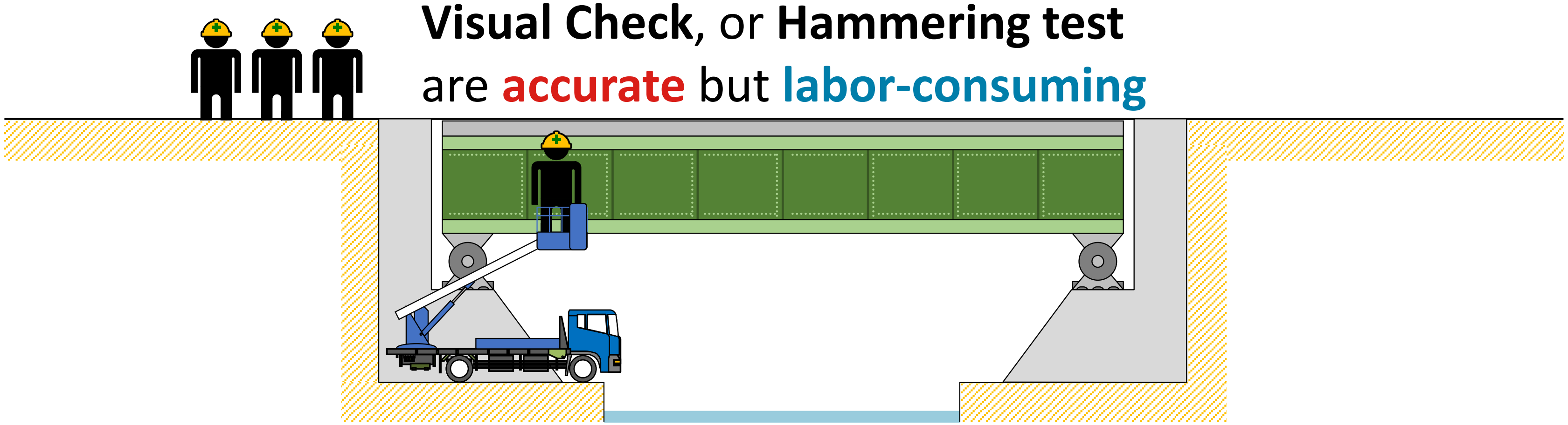
Our Theme

Easy and **Reliable** **Bridge Inspections**

(**Low-cost** & **Labor-saving**)
Anyone can do it

(**Objective**: ex) vibration-based SHM)
Same result no matter who does it

Visual Check, or **Hammering test**
are **accurate** but **labor-consuming**

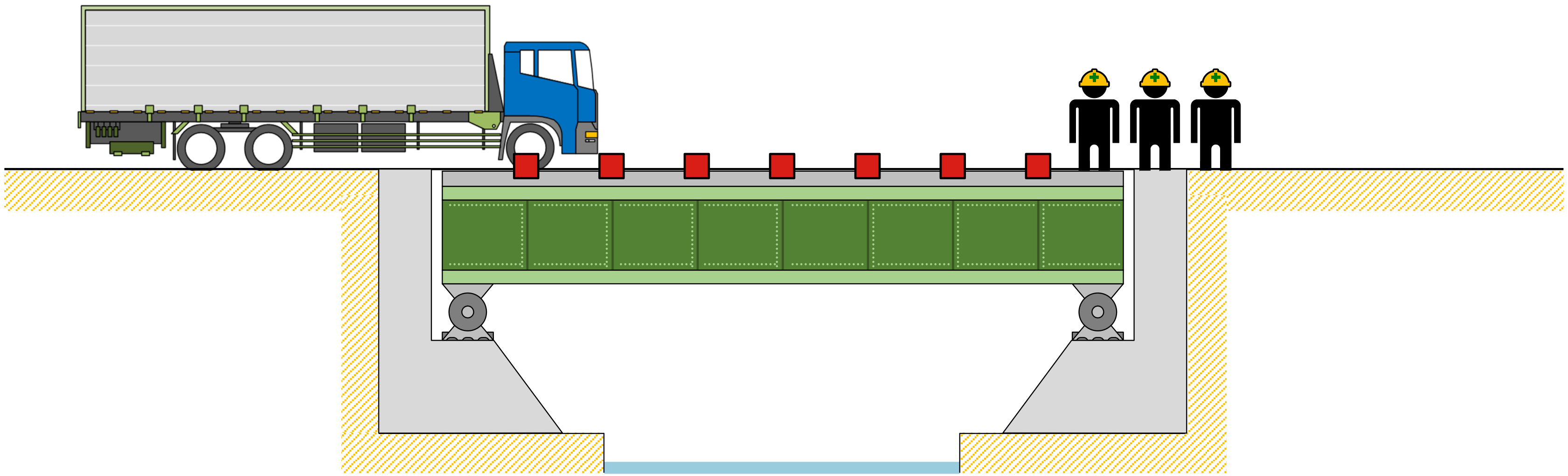


Target Bridges and Methodology

Vibration-based SHM for **Short-span Bridges**

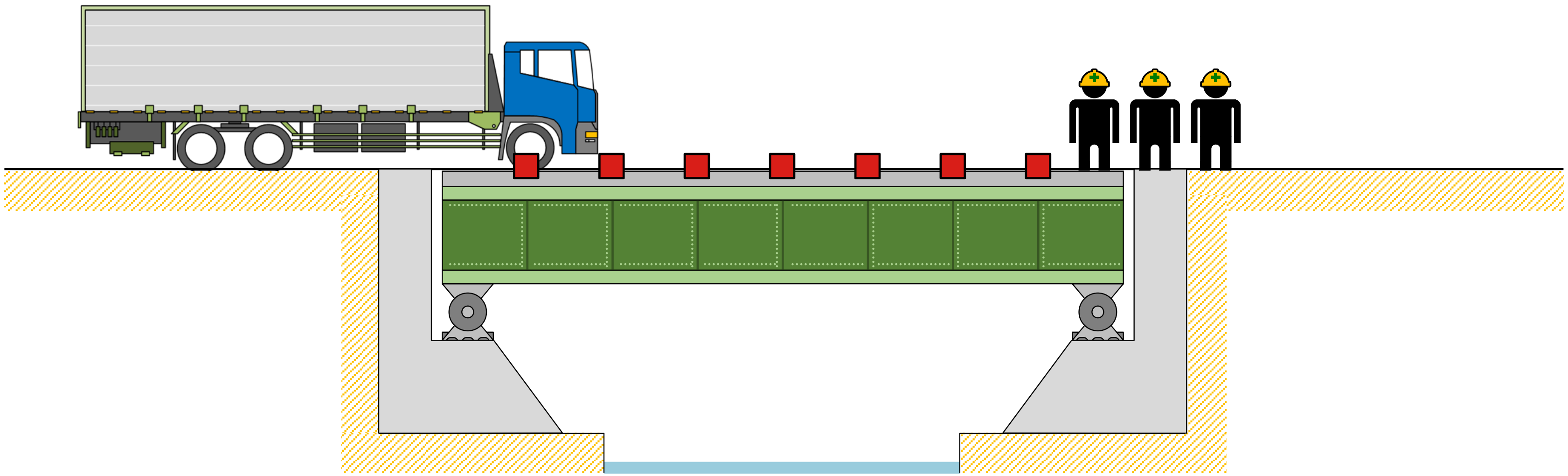
Sensors are installed on the bridges

More than **700,000** in Japan



Traffic-induced Vibration is predominant

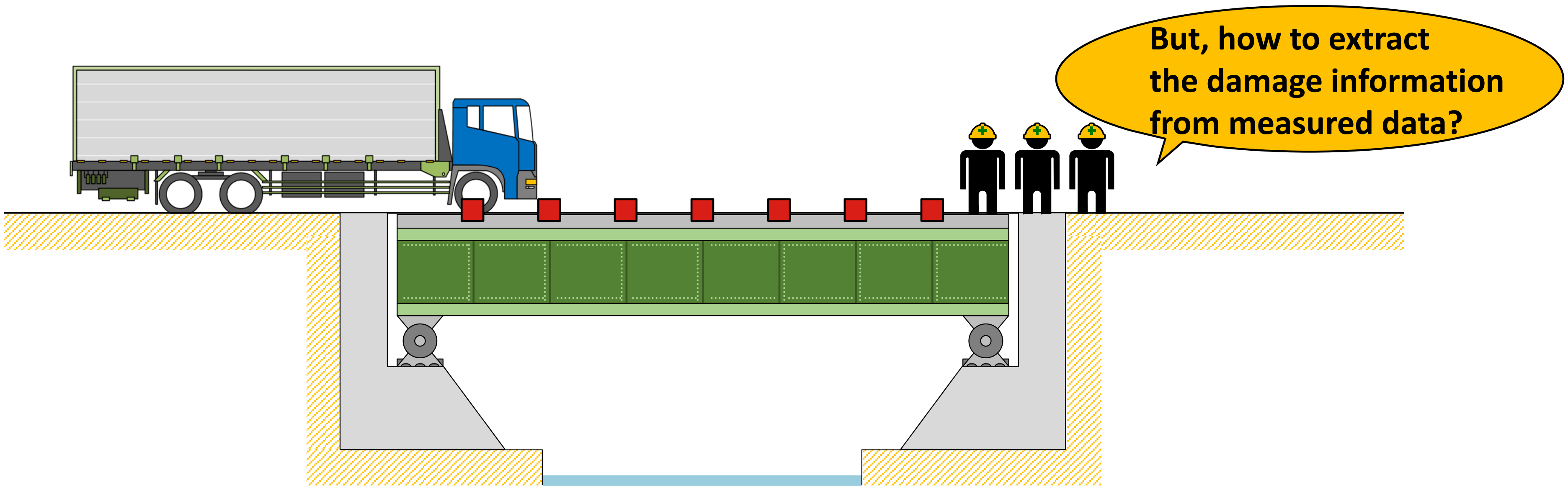
Predominant Frequencies and Mode Shapes do not match the Natural ones



Merit on using Traffic-induced Vibration

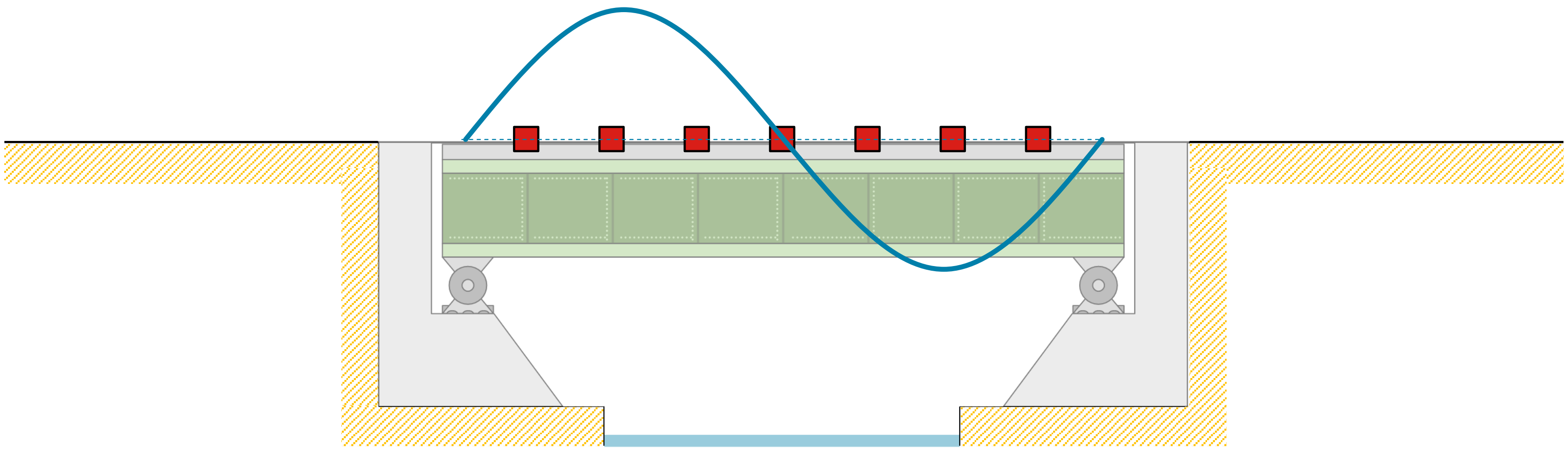
Damage Information may be easily found

Local damage usually affects the **higher order modal parameters**



Mode Shape is a good **Damage Indicator**

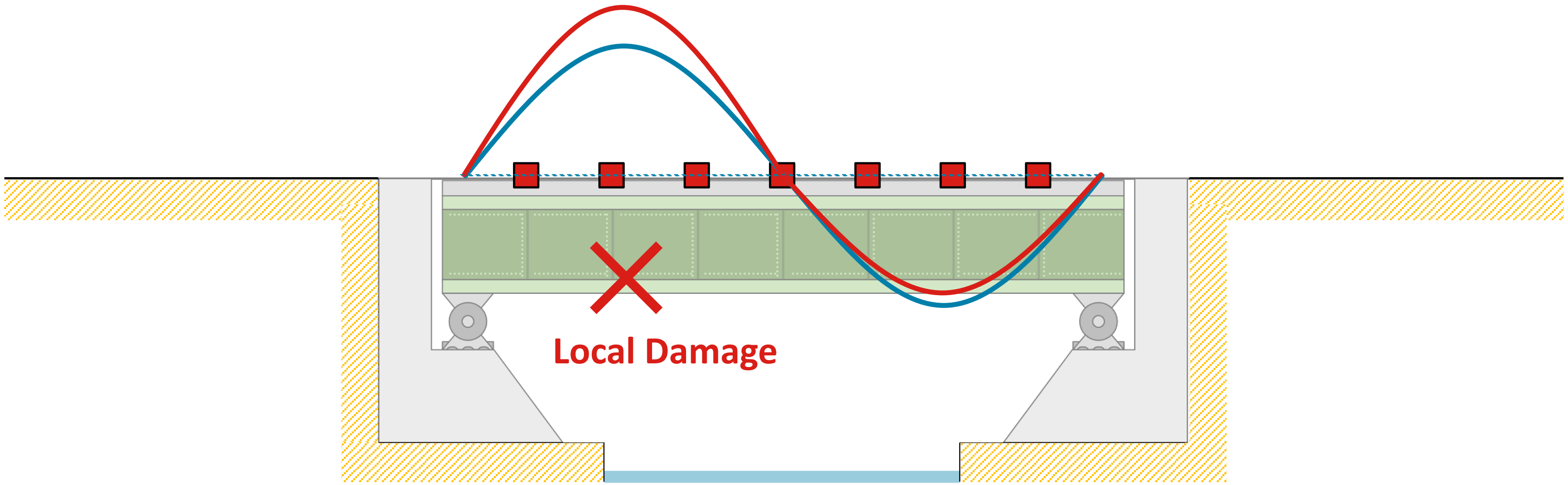
Defects locally reduce the flexural rigidity EI and mass ρA



Mode Shape is a good **Damage Indicator**

Defects locally reduce the flexural rigidity EI and mass ρA

and Mode Shape is **sensitive** to **local damages**



What is Mode Shape?

Mode Shape Matrix is **Orthogonal**

Equation of Motion

$$\mathbf{M}\ddot{\boldsymbol{y}} + \mathbf{K}\boldsymbol{y} = \mathbf{0}$$

Free vibration

$$\boldsymbol{y} = \boldsymbol{\alpha}_k \mathrm{e}^{j\omega_k t}$$

Mode Shape ($\boldsymbol{\alpha}_k$) Natural Frequency (ω_k)

$$-\omega_k^2 \mathbf{M}\boldsymbol{\alpha}_k \mathrm{e}^{j\omega_k t} + \mathbf{K}\boldsymbol{\alpha}_k \mathrm{e}^{j\omega_k t} = \mathbf{0}$$

$$[\mathbf{M}^{-1}\mathbf{K}]\{\boldsymbol{\alpha}_k\} = \omega_k^2 \{\boldsymbol{\alpha}_k\}$$

Eigen-value Decomposition

$$[\mathbf{M}^{-1}\mathbf{K}] = \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^{\mathrm{T}}$$

$$\mathbf{A}^{-1} = \mathbf{A}^{\mathrm{T}}$$

Mode Shape Matrix

$$\mathbf{A} = [\boldsymbol{\alpha}_1 \quad \cdots \quad \boldsymbol{\alpha}_n]$$

$$\boldsymbol{\Sigma} = \mathrm{diag}[\omega_1^2 \quad \cdots \quad \omega_n^2]$$

Natural Frequency

How to estimate?

Operational Modal Analysis is needed

To estimate the **modal parameters** from **transient responses**

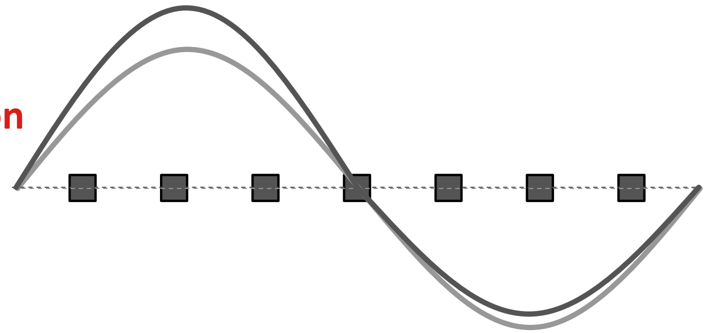
the **SVD** method

Singular Value Decomposition

SVD is like **Eigen-Value Decomposition**

often used for **Data-Compression**

Applicable to estimate
Bridge Mode Shape
but **No Consensus**



the **FDD** method

Frequency Domain Decomposition

FDD is like Cross-Power method

for mode shape and Frequency

Very Popular method
for BMS estimation
but **complicated**

SVD assumes that $\boldsymbol{q}(t)$ is **uncorrelated**

$$\boldsymbol{y}(t) = \mathbf{A}\boldsymbol{q}(t)$$

Diagonalization

$$E[\boldsymbol{y}(t)\boldsymbol{y}(t)^{\mathrm{T}}] = \mathbf{A}E[\boldsymbol{q}(t)\boldsymbol{q}(t)^{\mathrm{T}}]\mathbf{A}^{\mathrm{T}} = \mathbf{A}\mathbf{D}\mathbf{A}^{\mathrm{T}}$$

Assuming that $\boldsymbol{q}(t)$ is **uncorrelated**,
$\mathbf{D}$ becomes to be **diagonal**

FDD assumes that $\boldsymbol{q}(\omega)$ is **independent**

If $\omega = \omega_k$, the shape of $\boldsymbol{y}(\omega)$ matches the k-th mode shape

Diagonalization

$$\boldsymbol{y}(\omega)\boldsymbol{y}^{\mathrm{H}}(\omega) = \mathbf{A}(\omega)\boldsymbol{q}(\omega)\boldsymbol{q}(\omega)^{\mathrm{H}}\mathbf{A}(\omega)^{\mathrm{H}} = \mathbf{A}(\omega)\mathbf{S}_{\mathrm{FF}}(\omega)\mathbf{A}(\omega)^{\mathrm{H}}$$

cross-power spectrum

If $q_k(\omega)$ is not zero,
$q_l(\omega)$ ($l = 1\sim n, \text{except } k$) are zero

$\mathbf{S}_{\mathrm{FF}}(\omega)$ is called **Singular Spectrum**, of which peaks are estimated natural frequency

The k-th order estimated mode shape $\mathbf{A}(\omega_k)$ can be found by detecting k-th peak ω_k

Different Assumptions

SVD and **FDD** uses different assumptions

If the bridge is in the state of **free vibration** or **excited by white noise loads**, the results from SVD and FDD will be the **same**

Traffic-induced vibration **does not** satisfy this condition

the **Size of Difference** can be used to evaluate damage?

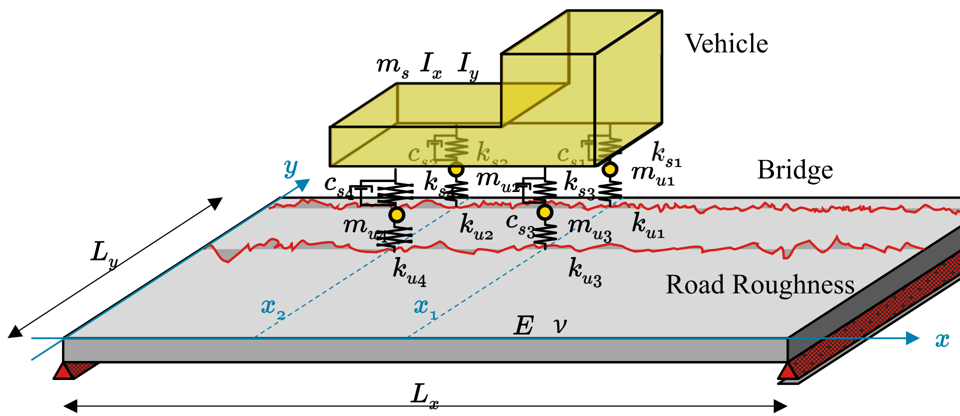
(Hybrid Method)

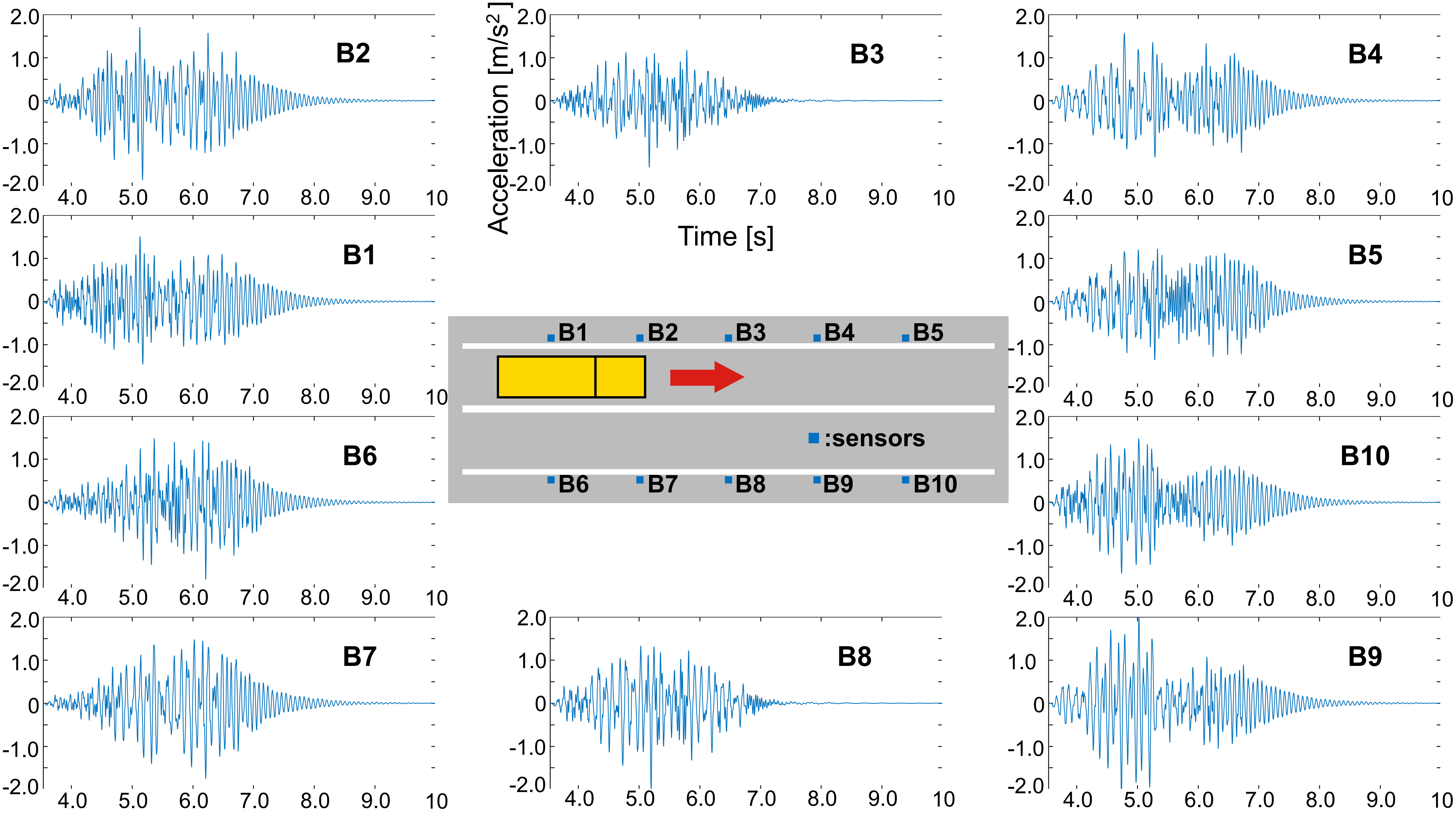
The Purpose of this Study

To **Compare** the **Mode Shapes** estimated by **SVD** and **FDD**

Similarity is evaluated by **MAC**

Bridge Vibration Data is simulated by numerical simulation

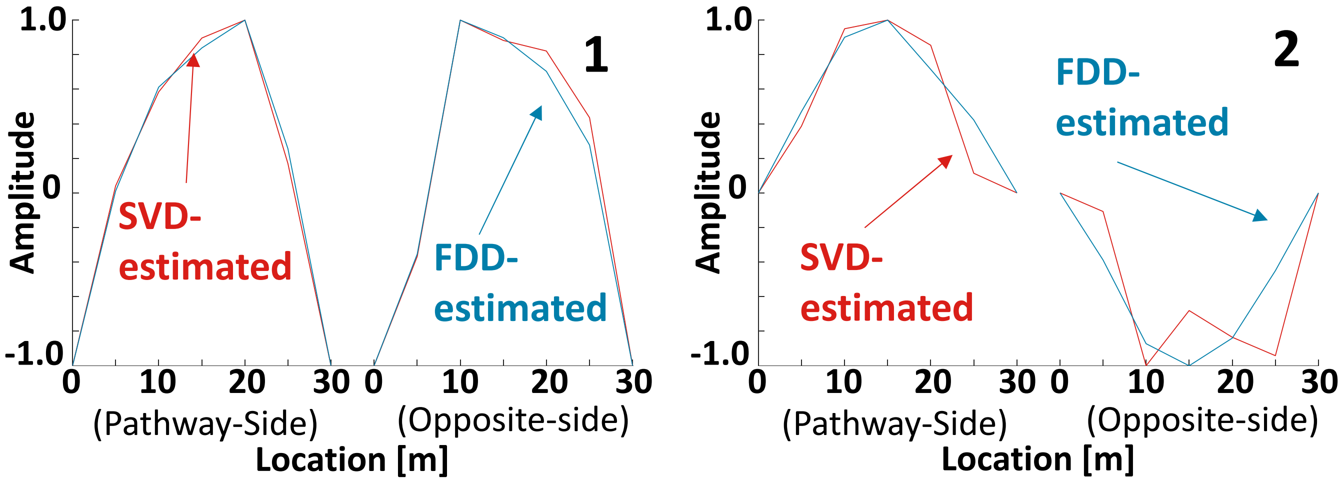




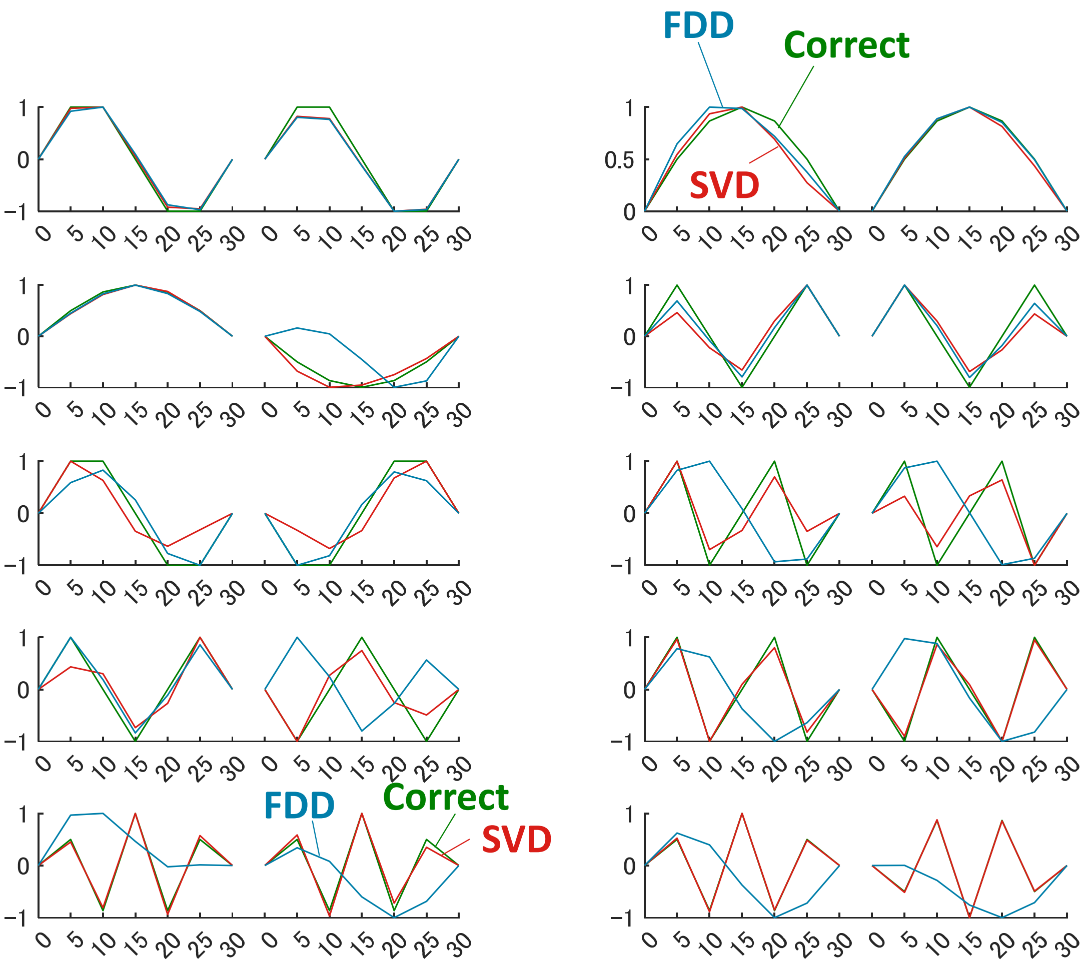
B2
B3
B4
B1
B5
B6
B10
B7
B8
B9
Acceleration [m/s^2]
Time [s]
B1
B2
B3
B4
B5
B6
B7
B8
B9
B10
:sensors

Estimated Bridge Mode Shapes (Low Order)

The **SVD** and **FDD**'s estimates are similar in Low Order



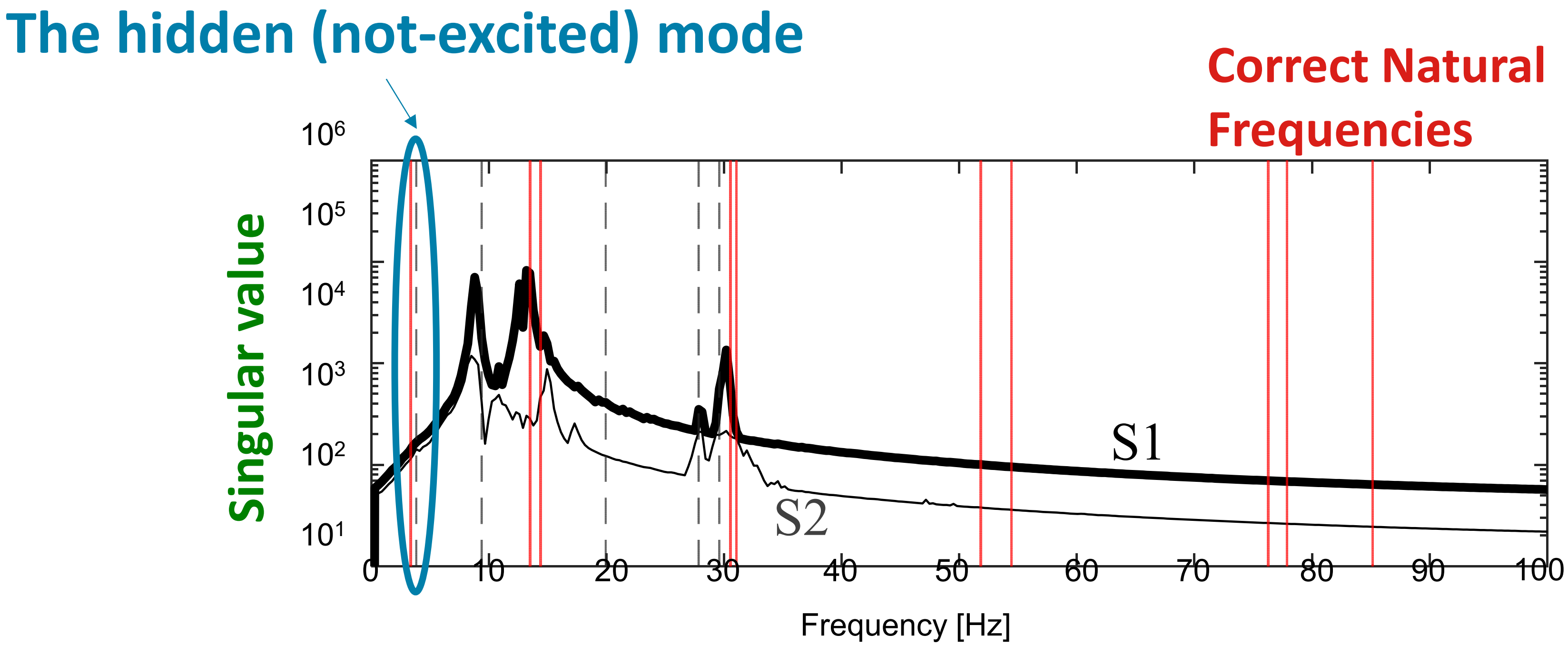
Estimated Bridge Mode Shapes



However, those in Higher Order are **much different**

Comparing them with **Correct values**
the accuracy of SVD is better

Singular Spectrum from **FDD**



SS of FDD cannot also find the natural frequencies that **SVD** can estimate

Conclusion

Traffic-induced Vibration is numerically simulated to compare the applicability of **SVD** and **FDD** to **Operational Modal Analysis**

SVD is not popular for Operational Modal Analysis
but shows good accuracy to estimate **Bridge Mode Shapes**
from **Traffic-induced Vibrations**

FDD is a popular method for Operational Modal Analysis
but it cannot estimate **the hidden (not-excited) modes**

SVD can estimate the hidden modes from the orthogonality of Mode Matrix