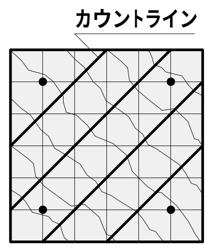
Calculation Procedures for Shear Stress vs Shear Strain

NOTATIONS (Unit system : N / mm^2 = MPa = 0.0981 kgf / cm^2, concerning only for calculating f_{cr} from f_c)

<panel>



t : panel thickness

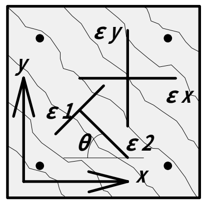
τ_{xy} : shear stress

f_y : y-direction stress (=0)

f_x : x-direction stress (=0)

γ_{xy} : shear strain

s_m : average crack interval (45 degree direction)
$=(150+250+150+250)\sqrt{2}$
/ number of cracks on count lines



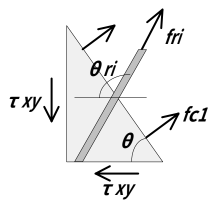
ε_1 : tensile principal strain

ε_2 : compressive principal strain

ε_x : x-direction strain

ε_y : y-direction strain

θ : angle between x-direction and compressive principal strain (clockwise as positive)



<i-th reinforcement> (i=1 to n, n=<10 is available)

E_{ri} : elastic modulus

f_{rui} : tensile strength or yield strength

f_{ri} : average stress

ε_{ri} : average strain

A_{ri} : sectional area

s_i : interval (pitch)

ρ_i : sectional area ratio = $\dfrac{A_{ri}}{s_i \cdot t}$

θ_{ri} : angle between x-direction and i-th reinforcement
(clockwise as positive $0^\circ \le \theta_{ri} < 180^\circ$)

θ_i : angle between tensile principal strain and i-th reinforcement
(clockwise as positive = $(\theta + 90^\circ) - \theta_{ri}$)

ρ_{ix} : effective ratio for x-direction = $\rho_i \left|\cos\theta_{ri}\right|$

ρ_{iy} : effective ratio for y-direction = $\rho_i \sin\theta_{ri}$

P_{rcri} : tensile force at crack

P_{rmi} : average force of reinforcement

f_{rcri} : tensile stress at crack

<concrete>

f_c : compressive strength (negative value)

ε_c : strain at compressive strength (negative value)

f_{cr} : crack strength

ε_{cr} : strain at crack strength

E_c : elastic modulus (calculating as $2 \cdot f_c / \varepsilon_c$)

f_{c1} : average tensile stress

f_{c2} : average compressive stress

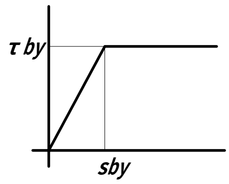
$f_{c2\max}$: compressive strength of cracked concrete (negative value)

<others>

n_i : elastic modulus ratio of reinforcement to concrete $= E_{ri} / E_c$

τ_{by} : yield bond stress

s_{by} : slippage at yield bond stress



Bond stress - sllipage

Step 1

Give ε_1 for calculating this procedure.

Step 2

Give θ in arbitrary.

Step 3

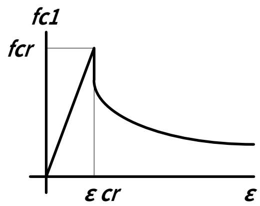
Give Σf_{riy} in arbitrary.

Step 4

Calculate f_{c1} (Modified Compression-Field Theory by Collins et al.).

$$\begin{cases} f_{c1} = \dfrac{2 \cdot f_c \cdot \varepsilon_1}{\varepsilon_c} & (\varepsilon_1 \le \varepsilon_{cr}) \\ f_{c1} = \dfrac{f_{cr}}{1 + \sqrt{200 \cdot \varepsilon_1}} & (\varepsilon_1 \ge \varepsilon_{cr}) \end{cases}$$

$$f_{cr} = 0.33\sqrt{-f_c} \qquad \varepsilon_{cr} = \frac{f_{cr} \cdot \varepsilon_c}{2 \cdot f_c}$$

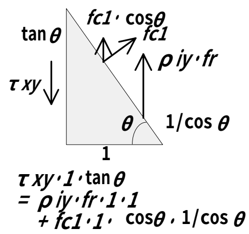


Tensile stress – strain of concrete

Step 5

Calculate τ_{xy}.

$$\tau_{xy} = \frac{f_{c1} + \Sigma\rho_{iy} \cdot \Sigma f_{riy}}{\tan\theta}$$

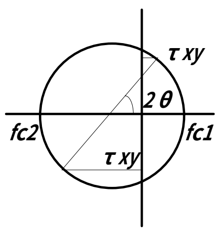


Equilibrium condition for y-direction

Step 6

Calculate f_{c2}.

$$f_{c2} = f_{c1} - \tau_{xy}(\tan\theta + 1/\tan\theta)$$



Mohr’s stress circle

Step 7

Calculate $f_{c2\max}$ from ε_1 (Kanakubo et al., 2000).

$$\frac{f_{c2\max}}{f_c} = -\frac{1}{0.95 \cdot \varepsilon_1 / \varepsilon_c} \le 1.0$$

Step 8

Judge for concrete failure.

If $f_{c2} > f_{c2\max}$, finish the procedure.

Step 9

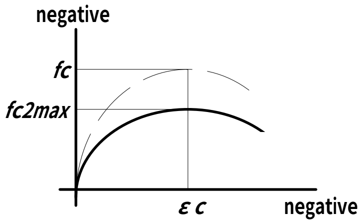
Calculate ε_2 (Modified Compression-Field Theory by Collins et al.).

$$\varepsilon_2 = \varepsilon_c \cdot \left(1 - \sqrt{1 - \frac{f_{c2}}{f_{c2\max}}}\right) \qquad \because \frac{f_{c2}}{f_{c2\max}} = 2\left(\frac{\varepsilon_2}{\varepsilon_c}\right) - \left(\frac{\varepsilon_2}{\varepsilon_c}\right)^2$$

Step 10

Calculate ε_y.

$$\varepsilon_y = \frac{\varepsilon_1 + \varepsilon_2 \cdot \tan^2\theta}{1 + \tan^2\theta}$$



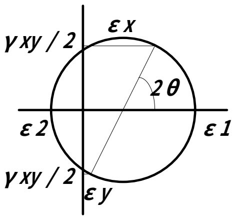
Compressive stress – strain for concrete

From Step 11 to 14, for each reinforcement (i=1 to n)

Step 11

Calculate θ_i.

$$\theta_i = (\theta + 90^\circ) - \theta_{ri}$$



Mohr's strain circle

Step 12

Calculate ε_{ri}

$$\varepsilon_{ri} = \frac{\varepsilon_1 + \varepsilon_2}{2} + \frac{\varepsilon_1 - \varepsilon_2}{2} \cdot \cos 2\theta_i$$

Step 13

Calculate f_{ri}.

$f_{ri} = E_{ri} \cdot \varepsilon_{ri}$ If $f_{ri} > f_{rui}$ in case of yield, $f_{ri} = f_{rui}$

Step 14

Calculate f_{riy} and f_{rix}.

$$\begin{cases} f_{riy} = f_{ri} \cdot \sin\theta_{ri} \\ f_{rix} = f_{ri} \cdot |\cos\theta_{ri}| \end{cases}$$

Step 15

Calculate Σf_{riy} and Σf_{rix}.

Step 16

Compare Σf_{riy} between Step 3 and Step 15. If these are not equal, give another Σf_{riy} and return Step3.

Step 17

Calculate ε_x.

$$\varepsilon_x = \varepsilon_1 + \varepsilon_2 - \varepsilon_y$$

Step 18

Calculate f_x.

$$f_{cx} = f_{c1} - \frac{\tau_{xy}}{\tan\theta}$$

$$f_x = f_{cx} + \Sigma\rho_{ix} \cdot \Sigma f_{rix}$$

Step 19

Check $f_x = 0$. If $f_x \neq 0$, then give another θ and return Step 2.

From Step 20 to 24, for each reinforcement (i=1 to n)

Step 20

Calculate s_{mi} .

$$s_{mi} = s_m \frac{\sin(135^\circ - \theta)}{|\cos\theta_i|}$$

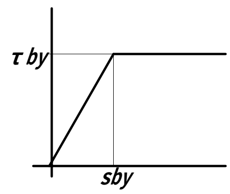
However, in case of $\theta_i = 90^\circ$, no calculating needs.

Step 21

Calculate P_{rcri} .

$$P_{rcri} = \frac{(1+n_i\rho_i)\cdot P_{rmi}\cdot s_{mi}}{n_i\rho_i\cdot s_{mi} + \dfrac{2}{k\cdot\tanh(k\cdot s_{mi}/2)}}$$

$$P_{rmi} = f_{ri}\cdot A_{ri} \qquad k = \sqrt{\frac{(1+n_i\rho_i)\cdot\tau_{by}}{E_{ri}\cdot A_{ri}\cdot s_{by}}}$$



Bond stress - slippage

Step 22

Calculate τ_{cri} .

$$\tau_{cri} = \frac{k\cdot P_{rcri}\cdot\sinh(k\cdot s_{mi}/2)}{(1+n_i\rho_i)\cosh(k\cdot s_{mi}/2)}$$

Step 23

If $\tau_{cri} \le \tau_y$, go to the next step. If $\tau_{cri} > \tau_y$, calculate P_{rcri} .

$$P_{rcri} = \frac{P_{rmi}\cdot s_{mi} + \tau_y\cdot l_{yi}^2 + 2E_{ri}\cdot A_{ri}\cdot s_y/(1+n_i\rho_i)}{2l_{yi} + \dfrac{s_{mi}-2l_{yi}}{1+1/n_i\rho_i}}$$

$$\frac{P_{rcri}-(1+n_i\rho_i)\tau_y\cdot l_{yi}}{E_{ri}\cdot A_{ri}} + \frac{k\cdot s_y}{\tanh(k\cdot s_{mi}/2 - k\cdot l_{yi})} = 0$$

Step 24

Calculate f_{rcri} , judge the rupture of reinforcement.

$$f_{rcri} = \frac{P_{rcri}}{A_{ri}}$$

If $f_{rcri} > f_{rui}$ in case of rupture, finish the procedure.

Step 25

Calculate γ_{xy} .

$$\gamma_{xy} = \frac{2(\varepsilon_x - \varepsilon_2)}{\tan\theta}$$

Finishing for one procedure. For next ε_1 , go to Step 1.