Japan Concrete Institute TC Activities on Bond Behavior and Constitutive Laws in RC

(Part 3: Application of Constitutive Laws for FEA)

T. Kanakubo
University of Tsukuba, Tsukuba, Japan

Y. Sato
Kyoto University, Kyoto, Japan

Y. Uchida
Gifu University, Gifu, Japan

K. Watanabe
Railway Technical Research Institute, Kokubunji, Japan

H. Shima
Kochi University of Technology, Kochi, Japan

ABSTRACT: Finite Element Analysis (FEA) is the one of the effective simulating methods to represent the behavior of Reinforced Concrete (RC) members and structures. The Japan Concrete Institute Technical Committee on Bond in Concrete Structures has conducted the following two main research programs in order to discuss the effective usages of bond models in FEA. The one is the meso-scale analysis using FEA and Rigid Body Spring Model (RBSM) by the modeling of bearing between lugs of reinforcements and concrete, and the other is the actual applications of bond models into FEA of RC members. The objectives of these programs are to understand the bond mechanisms of reinforcements and bond stress - slip relationships, and to abstract the problems of bond-link model in FEA.

1 INTRODUCTION

Brittle fractures of reinforced concrete members include bond fracture and anchorage fracture. The full understanding of bond fractures is, however, very difficult owing to the interactions of microscopic stress transmission, the constitutive law of each material, linked fractures at the macroscopic level, etc. In recent years, the Finite Element Analysis (FEA) has become a useful tool to determine the behavior of reinforced concrete members. The FEA is, however, significantly affected by the model of bond behavior, specifically the preconditions in a bond-softening region considerably affect the accuracy of the solution. The Japan Concrete Institute Technical Committee on Bond in Concrete Structures carried out research activities from 2009 to 2011 aiming to grasp the bond behavior on a microscopic scale, thus to review the information relating to bond behavior and to make a proposal for using the information specifically through determination of the bond constitutive law in FEA aimed at its practical application.

In this paper, the results of the following two main research programs are reported. The one is the meso-scale analysis using FEA and Rigid Body Spring Model (RBSM) by the modeling of bearing between lugs of reinforcements and concrete, and the other is the actual applications of bond models into FEA of RC members. The objectives of these programs are to understand the bond mechanisms of reinforcements and bond stress - slip relationships, and to abstract the problems of bond-link model in FEA.

2 MESO-SCALE ANALYSIS OF BOND BEHAVIOR

Studies began as early as the '80s to determine the bond behavior through direct modeling analysis of mechanical mating between nodes of a reinforcing bar and concrete, which is the essence of the bond between a deformed reinforcing bar and concrete. (IngarrFea et al. 1984, Kokusho et al. 1981, Reinhardt et al. 1984, etc.) According to the analysis, the τ-s relation is not regarded as a constitutive law, and the bond phenomenon is reproduced as a result of the propagation of fracture of concrete peripheral to the reinforcing bar, such as the internal cracks occurring from the front end of a node and the plasticizing of concrete at the front side of the node. Consequently, it is expected that the analytical results give a macroscopic bond model.
For the analysis of a specimen or member modeled to the size of the node of a reinforcing bar, the problem to be solved becomes large scale so that a large computational capacity is required. Recently, however, even a problem of several tens of thousands of degrees of freedom can be computed at a practical level, and several analyses have already been reported. Examples of analysis by RBSM and FEA are described.

2.1 Example of RBSM analysis

RBSM is a model which can directly express the crack phenomena of opening and shear, and is an analytical method suitable for analyzing fracture phenomena accompanied by the propagation of cracks. Muto et al. (2004) conducted analysis with a tensile bond specimen having a circular cross section, as illustrated in Figure 1, by modeling to the size of a lug of a reinforcing bar. The nominal diameter of reinforcing bar is 32 mm, lug spacing and lug height is 22 mm and 2.2 mm, respectively.

Figure 2 shows the relation between the average stress and the average strain on the entire member. The analytical result agreed well with the values derived from the conventional empirical equation. Figure 3 shows the analytical result for crack propagation. Also, the ultimate crack distance (macroscopic cracks penetrating the cross section) agreed well with the values derived from the conventional empirical equation. Since modeling to the size of the lug of a reinforcing bar allows us almost to reproduce the macroscopic bond behavior of a deformed reinforcing bar, it is expected that it will be possible to evaluate the bond mechanism analytically.

Figure 4 illustrates the tensile bond specimen to perform FEA for meso-scale analysis. The specimen has a cross section of 200 mm \times 250 mm with a deformed reinforcing bar of 19 mm (D19) in diameter. In the experiment for this specimen, the cracks were observed with an interval of about 500 mm. Therefore, the portion of 1000 mm length with quarter part of the cross section is modeled for FEA.

Figure 5 shows the FE model of concrete and reinforcing bar, and boundary elements between concrete and reinforcing bar. The solid element is used for concrete and reinforcing bar. The number of nodes is 31959, elements is 26400, and the numbers of boundary elements is 1600. The perfect elastic – plastic model for compression side and the softening model for tension side of concrete are adopted. The compressive and tensile strength of concrete is 25.3 and 2.53 MPa, respectively. The constitutive laws
for the boundary elements are shown in Figure 6. The enough-lower stiffness for axial direction surface is set for the boundary element A.

Figure 7 shows the relationship between stress of reinforcing bar and whole elongation comparing experimental results and FEA. Crack stress differs between experiment and FEA. The reason is considered that the shrinkage stress occurs in the specimen. However outline of FEA curve agrees with the experimental result. Figure 8 shows bond stress versus relative slip curves between reinforcing bar elements and concrete elements. The curves are plotted by the positions from the loaded end of the specimen. Except for the curves of 0.5D, 1.5D (D = bar diameter) curves, curves behave similarly. After the crack occurs, the bond stresses decrease and become showing negative bond stresses (opposite direction). The differences of 0.5D and 1.5D curves from other
curves are due to cone shaped crack which takes place at the end of specimen.

3 ACTUAL APPLICATIONS OF BOND MODELS INTO FEA

The results of two series of FEA are reported in this chapter. Both FEAs are conducted to study the effect of bond models on the behavior of RC members comparing the experimental results. At first, an uniaxial tensile test is investigated. This is a basic experimental method to investigate the bond characteristics between a reinforcing bar and concrete.

3.1 FEA of uniaxial tensile test for prism specimen

Engineers and researchers often use the bond link elements or other similar elements in FEA to express the bond slip between reinforcing bar and concrete. On the other hand, the bond behavior is indirectly expressed by the tension stiffening model of concrete in case of smeared crack model. The tension stiffening model is boldly modeled of tensile stress of concrete transmitted by the bond. If the bond link elements and tension stiffening model are simultaneously used, it is anxious that the effect of bond is double counted as shown in Figure 9. In this section, “double counted effect” is investigated by the FEA using both bond link elements for discrete reinforcing bar and tension stiffening model for concrete comparing with the experimental results.

3.1.1 Experiments and target specimens

The target specimens for FEA are uniaxial tensile prism specimens tested by Kanakubo et al. (2009) with square section of 80 mm, 100 mm and 120 mm. A deformed bar of 10 mm (D10) in diameter is arranged at the center of the section, and the reinforcement ratios (ρ) are 1.23 %, 0.79 % and 0.55 %, respectively. The length of specimen is 1600 mm. The four-node plane elements, two-node truss elements and four-node bond link elements are used.

Before the investigation of double counted stress, the tension stiffening curve is modeled by an integration calculation based on the assumption of bond stress – slip relationship as shown in Figure 10. The

![Figure 9. Effect of tension stiffening model for uniaxial tensile specimen. Double counted stress is included by tension stiffening and bond link model.](image)

![Figure 10. Assumption of bond stress – slip curve.](image)

![Figure 11. Comparison of tensile load – elongation curve between test results and analytical results.](image)
calculation method is the integration of the one-dimensional elements under the force equilibrium condition and compatibility condition including slip between reinforcing bar elements and concrete. The calculated region is between cracks. If the tensile stress of concrete elements becomes bigger than the tensile strength of concrete, new crack takes place at the center of the region, and the length of calculated region is set to half length. The calculation is continued until the new crack does not occur. The results of calculation are shown in Figure 11 comparing with the test results. The shape of tensile load–elongation curves from the calculation almost agrees with the test results. Figure 12 shows the analytical results of tensile concrete stress which is obtained by the subtraction of the force of reinforcing bar from the whole tensile load, and its modeling by multi-linear model.

3.1.2 FEA modeling and results
Two series of FEAs by smeared model and discrete model are performed. Figure 13 shows the modeling for both cases. In the smeared-based analysis, the tension stiffening model shown in Figure 12 is adopted for all concrete elements. The bond link model is also used. The sizes of elements range from...
12.5 mm to 1600 mm (8 cases). In the discrete-based analysis, only the central element behaves under the tension stiffening model. The other elements are set to show elastic manner without bond link. After the cracking at the central element, the half length model is newly adopted. The same progress is continued until no crack occurs.

The analysis results are shown in Figures 14 and 15 for the smeared-based model and the discrete-based model, respectively. From the results of the smeared-based model, it is observed that FEA results show the almost similar curves with tension stiffening model shown in Figure 12. This means that “double count” of tension stiffening effect does not occur. In case of smeared model, all elements for concrete show the same strain, so that bond link does not work. The use of combination of smeared concrete element, discrete bar element and tension stiffening model is effective only for expressing the large scale of slip of reinforcing bar such as slip out of bar from concrete. From the results of the discrete based model, the obtained curves show almost the same tendency with the tension stiffening model. However, the crack spacing by the analysis (25 mm
in case of $\rho=1.23\%$ differs from the experimental result (100 mm). This difference is due to the “double count” of tensile stress. Thus, the effect of double count appears as the difference of crack spacing.

3.2 FEA of RC beams failed by bond splitting

FEAs for RC beam specimens having a small shear span ratio ($a/d$) under an anti-symmetrical bending moment are conducted to investigate the adaptability of bond models. It is known that small span ratio beams under anti-symmetrical bending moment show the cracks along the longitudinal bars with splitting crack of concrete. This failure is called as bond splitting failure. The objective is to solve the behavior of bond splitting beams considering bond stress transmission mechanism between reinforcing bar and concrete. The sectional size of beams is 400 mm in effective height and 300 mm in width. The shear span ratios are 1 (DB408), 1.5 (DB608) and 2 (DB808), respectively. Four deformed longitudinal bars of 29 mm in diameter are arranged while deformed stirrups of 13 mm in diameter are used with 100 mm spacing. The concrete compressive strength ranges from 28.9 MPa to 29.3 MPa.

3.2.1 Analysis by DIANA

The analysis uses the commercially available FEA software DIANA, and applies the model of smeared cracks - discrete reinforcing bars. The model applied to the concrete is the fixed crack model in which the shear stress - shear strain relation is used considering compatibility with the loading experiment. The nonlinear constitutive law for the concrete adopted the Thorenfeldt model and the Hordijk model, which considered the fracture energy. To express the interaction between the concrete and the longitudinal reinforcing bar, a trial analysis was performed under the condition that the bond characteristic is assumed as perfect bond, using the $\tau$-$\delta$ curve proposed by Shima et al. (1987) and Suga et al. (2001).

Figure 16 shows examples of analytical results. In the analysis for DB408, many slant cracks occur at the center of beams. Since the yield of stirrup is observed at the peak load, the analysis presumably can pursue the failure mode similar to experiment. On the other hand, for DB608 and DB808, bending shear cracks and cracks along the longitudinal direction of the reinforcing bar occur. However, many slant cracks are observed at the center of the beam, showing a crack pattern different from that observed in the experiment.

3.2.2 Analysis by FINAL

FEA is conducted applying the bond model of Goto, Morita and Fujii (1981) to four-node bond link elements, and coercive bond splitting cracks. The bond model is a multi-linear curve expressing the bond behavior which accompanies the bond splitting strength and slip based on many experimental results. In FEA, the simplest method for expressing the deterioration of bond is to insert the bond link between the reinforcing bar elements and the concrete elements. However, the deteriorating of bond which actually propagates around the reinforcing bar in three-dimension is not modeled by bond link. In this FEA, a user subroutine to cause coercive bond splitting cracks linking to the Goto, Morita and Fujii model is applied to the bond link.

Figure 17 shows the relationships between the shear force and the displacement and the crack pat-
In the shear force-displacement relationships, the thin lines represent the test results, the thick lines the analyses without the splitting cracks, and the dotted lines the analyses with the splitting cracks. Since the splitting cracks appear after the peak load, the pre-peak behaviors are the same whereas the post-peak behaviors differ. The differences, however, are not significant in the shear force-displacement relationships. Regarding the cracks at the peak loads, the experiments showed regions without the cracks near the center of the beam. In the analysis, however, cracks are generated over the entire span.

In the experiment, before growing the shear cracks over the entire region of the span, splitting cracks were presumably generated along the longitudinal bars, thus a region of no cracks remained. The objective of introducing splitting cracks in the analysis is to reproduce the above phenomenon. As illustrated in Figure 17, however, many shear cracks have already formed at the point of reaching maximum load, and an effect of suppressing shear cracks by introducing splitting cracks is not observed.

4 CONCLUSIONS

In the meso-scale analyses of the reinforcing bar and surrounding concrete, a possibility is found to understand the bond behavior and the bond models by RBSSM and FEA. By the FEA of uniaxial tensile test of RC prism, the "double count" tensile stress by adopting both the tension stiffening model and the bond link element is discussed. In the FEA for the bond splitting beams, it is difficult to simulate the crack patterns caused by the bond splitting although the shear force-displacement behaviors are well simulated.

5 ACKNOWLEDGEMENT

The authors fully acknowledge the members of Japan Concrete Institute Technical Committee on Bond in Concrete Structures for their dedicated activities.

6 PREFERENCES

Ingarffea, A.R. et.al., 1984: Fracture Mechanic of Bond in Reinforced Concrete, ASCE Structural Division, Vol.110, No.4, pp.871-890