Proceedings of 2004 IEEE/RSJ International Conference on
Intelligent Robots and Systems
September 28 - October 2, 2004, Sendai, Japan

General-Purpose Expression of Structural Connectivity in the Parallel Solution Scheme of Inverse Dynamics

Daigoro Isobe
Graduate School of Systems & Information Engineering,
University of Tsukuba
Tsukuba-shi, Ibaraki 305-8573, Japan
isobe@kz.tsukuba.ac.jp

Atsushi Yagi
Graduate Student
University of Tsukuba
Tsukuba-shi, Ibaraki 305-8573, Japan

***Abstract*— The main purpose of this study is to deal with a sudden change of dynamics in link systems, not by using integrated systems of different software, but by using a single solution scheme based upon a single theory. An algorithm for the general-purpose expression of structural connectivity is developed and implemented into the parallel solution scheme, which was previously proposed and successively applied to the feed-forward control of link mechanisms under various boundary conditions. The algorithm expresses the connectivity of link members explicitly, regardless of the structural complexity. The parallel solution scheme calculates the inverse dynamics of link systems with equations of motion expressed in the dimension of force. It enables us to obtain numerical torque values in parallel by using a matrix-form equation separated into terms of different parameters. Therefore, the connectivity of link members can be expressed explicitly by one of the matrices, the member length matrix. We describe the forming process of the matrix and verify the validity of the calculated torque values, by presenting simple numerical results and experimental results for complex systems such as multibranch link systems. It is confirmed that a sudden structural change of link systems is dealt with only by a revision of input data, which makes it highly reliable in fail-safe systems.**

I. Introduction

Recently, demands on motion robots have increased greatly, such as that they move more quickly and in more complicated ways. Such demands may lead to the production of structurally complicated systems, of which the dynamic equations will inevitably become difficult for general users to derive (see Fig. 1). Generally, the dynamic equations are derived by the Newton-Euler method or the Lagrangian method, in relative polar coordinates and in the dimension of torque. The equations

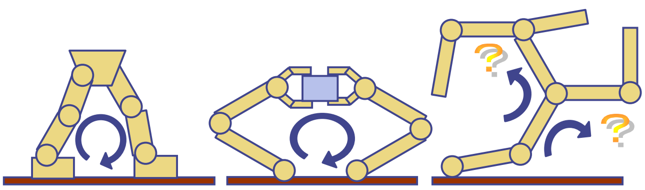


Figure 1 Cases which are difficult to handle for general users

supply exact solutions in a short calculation time. However, we require a special assumption to derive the equations, for closed-loop or other link systems with complicated structural connectivity [1][2]. This is due to the inclusion of interdependent variables between the constituting links in the dynamic equations. Therefore, a sudden change of dynamics is generally dealt by an integrated system of different algorithms or dynamic equations.

Nakamura and Yamane developed a computational algorithm for the inverse and forward dynamics of open and closed kinematic chains, which can be applied seamlessly against the motions of any rigid link systems without switching among algorithms [3]. However, the scheme requires the virtual cutting of the kinematic chain when applied to closed-loop chains, along with the computation of Jacobian matrices. On the other hand, Isobe developed a completely new scheme for calculating inverse dynamics by using a finite element approach [4]. The scheme is named the parallel solution scheme, since it computes nodal forces in parallel by using the equations of motion expressed in the dimension of force, and converts these to torque by using a matrix-form equation separated into individual terms of nodal forces, transformation between coordinates, and member length. Therefore, the scheme can not only deal with open- and closed-loop link systems independently, it can also deal seamlessly with those that gradually change their forms and dynamics. There is also no need to revise the basic numerical algorithm of the scheme, regardless of the stiffness of the constituting link member, whether it is rigid or flexible [5].

In this paper, we describe an algorithm for the general-purpose expression of structural connectivity, using the member length matrix in the parallel solution scheme. The matrix contains the member length information in the components, and expresses the structural connectivity of the link system by the components' configurations. The connectivity between two links relies only upon whether a link is mechanically dependent (or supported) on another, or vice versa. By using this algorithm, complex systems such as multibranch link systems can be expressed explicitly, simply by feeding the connectivity data into the inverse dynamics computation program. The parallel solution scheme with the proposed algorithm is verified by comparing a simple numerical result with the exact solution, and by carrying out some feed-forward control experiments on a structure-varying link system.

II. Parallel Solution Scheme

A. Comparison with Schemes Using Dynamic Equations

Dynamic equations for link systems are generally derived by the Newton-Euler method or the Lagrangian method. In a summarized expression, the equations are written as

$$\{\tau\} = [M(\theta)]\{\ddot{\theta}\} + \{V(\theta,\dot{\theta})\} + \{G(\theta)\}, \quad (1)$$

where $\{\tau\}$ is the torque vector, $[M]$ the inertial force matrix, $\{V\}$ the centrifugal force and Coriolis force term vector, and $\{G\}$ the gravity force term vector. θ, $\dot{\theta}$, and $\ddot{\theta}$ within the parentheses are the relative variables of the angle, the angular velocity and the angular acceleration between each link, respectively. All of the parameters in the equation relate to each other since they are derived in relative polar coordinates and in the dimension of torque. Therefore, most parts of the equations must be revised when the structural configuration of the link system is changed. Furthermore, special treatment must be adopted for the derivation of the equations for complex systems such as closed-loop or multibranch link systems.

On the other hand, torque values are calculated by using the following equation in the parallel solution scheme [4];

$$\{\tau^n\} = [L^n][T^n]\{P^n\}, \quad (2)$$

where $\{\tau^n\}$ is the torque vector, $\{P^n\}$ the vector related to nodal forces, $[T^n]$ the transformation matrix between global and elemental coordinates, and $[L^n]$ the member length matrix. The suffix '*n*' on the upright indicates the total number of links. The nodal forces are evaluated at an absolute Cartesian coordinate, and in the dimension of force. The equation is completely separated into terms of different parameters, and each matrix expresses the essence of the modeled link system such as member lengths and connectivity, initial and transition of coordinates and joint angles, and amount of applied forces at each joint. Therefore, it has high expansibility and flexibility, and it can be applied to complex link systems without difficulty.

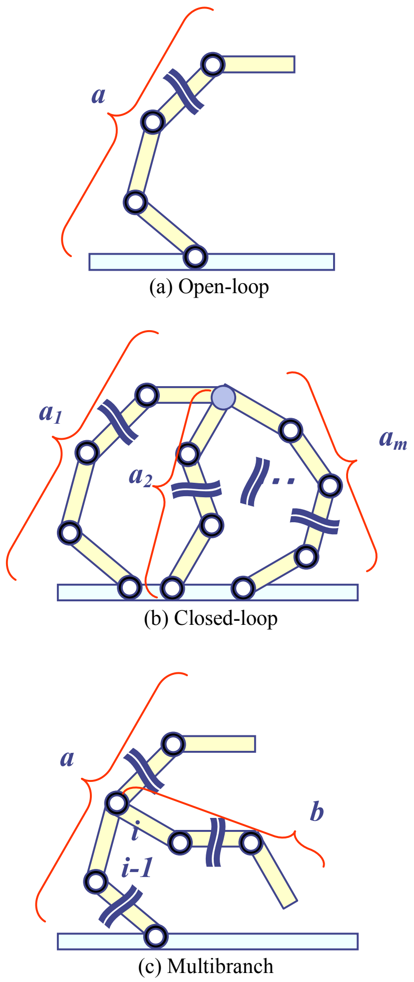
An incremental nodal force vector $\{\Delta f\}$, required for the link system in motion between time t and $t+\Delta t$, is derived by the following equation [4];

$$\{\Delta f\} = \{R\}_t - \{F\}_t + [M]\left(\frac{1}{\beta\Delta t^2}\{\Delta u\} - \frac{1}{\beta\Delta t}\{\dot{u}\}_t - \left(\frac{1}{2\beta} - 1\right)\{\ddot{u}\}_t\right), \quad (3)$$

where $[M]$ is the total mass matrix, $\{F\}$ the external force vector, $\{R\}$ the internal force vector, $\{\Delta u\}$ the incremental displacement vector, $\{\dot{u}\}$ the velocity vector, and $\{\ddot{u}\}$ the acceleration vector. β is the integral parameter for Newmark's β method [6], a widely used time integration scheme. The operation distance between each incremental step calculated from a target trajectory is used as an input for $\{\Delta u\}$. The velocity and acceleration vectors can also be given directly as input data, however, we used Newmark's β method (δ =1/2, β=1/4) for calculating the values in this paper. The values of the internal force vector $\{R\}$ will all actually become zero, since the consideration of the deformation of link members in the inverse dynamics calculation process is not required. The external vector includes information such as dead loads and other additional forces acting at nodal points. Each corresponding term in the incremental nodal force vector is then substituted into (2) to constitute the vector $\{P^n\}$. Although the scheme requires incremental calculation, it has been confirmed in previous works that the calculation time in actual control is suppressed to a practical value. Further information regarding the calculation process can be found in references [4] and [5].

B. Expressing Structural Connectivity by Member Length Matrix

The member length matrix $[L^n]$ in (2) contains components in the dimension of length, which play roles in converting the dimension from force to torque. Also, the configuration in the matrix expresses the structural



(a) Open-loop

(b) Closed-loop

(c) Multibranch

Figure 2 Various system configurations

connectivity of the link system. For example, the structural connectivity of an open-loop link system constituted with a link members, as shown in Fig. 2(a), can be expressed by an upper triangular matrix as follows:

$$\left[\ L^a\ \right] = \begin{bmatrix} L_1 & L_2 & \cdots & L_{a-1} & L_a \\ & L_2 & \cdots & L_{a-1} & L_a \\ & & \ddots & \vdots & \vdots \\ & 0 & & L_{a-1} & L_a \\ & & & & L_a \end{bmatrix}, \tag{4}$$

where each component $L_i (i = 1, \cdots, a)$ is a member length matrix for link No. i , containing components on member length, distance between center of gravity and nodal points, etc [4]. In the case of dealing with a closed-loop link system as shown in Fig. 2(b), where a_1, a_2, ... , a_m links (a_1 + a_2 + ... + a_m = n) are connected, the member length matrix can be expressed as follows:

$$\left[\ L^n\ \right] = \begin{bmatrix} L^{a_1} & & & & \\ & L^{a_2} & & 0 & \\ & & \ddots & & \\ & 0 & & L^{a_{m-1}} & \\ & & & & L^{a_m} \end{bmatrix}, \tag{5}$$

where $L^{a_1}, L^{a_2}, \cdots, L^{a_m}$ are each of the upper triangular matrices expressed as (4). Namely, serial structures to which the torque values should be added, are expressed as (4). Parallel structures to which the values should not be added, are expressed by a matrix with the upper triangular matrices placed on the diagonal, as in (5). For another example, the multibranch link system shown in Fig. 2(c), where a b-link open-loop system branches from the i-th joint in an a-link system, can be expressed as follows:

$$\left[\ L^n\ \right] = \begin{bmatrix} L_1^a & \cdots & L_i^a & \cdots & L_a^a & L_1^b & \cdots & L_b^b \\ & \ddots & \vdots & & \vdots & \vdots & & \vdots \\ & & L_i^a & \cdots & L_a^a & L_1^b & \cdots & L_b^b \\ & & & \ddots & \vdots & & 0 & \\ & & & & L_a^a & & & \\ & & & & & L_1^b & \cdots & L_b^b \\ & 0 & & & & & \ddots & \vdots \\ & & & & & & & L_b^b \end{bmatrix}. \tag{6}$$

Basically, the matrix consists of two independent upper triangular matrices that indicate two open-loop links. To add up the torque values in the branch to the supporting link, the first row components of the b-link system are all duplicated into the i-th row (which indicates a branching joint) and the former rows.

The flow chart of the construction algorithm of the member length matrix is shown in Fig. 3. A supported link and a supporting link between two connected links are indicated as the child link and the mother link, respectively. The flow is summarized as follows.

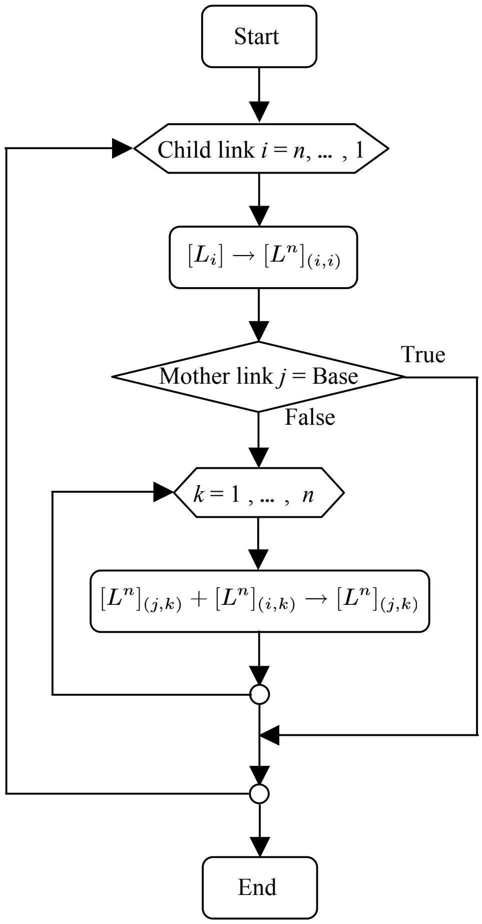
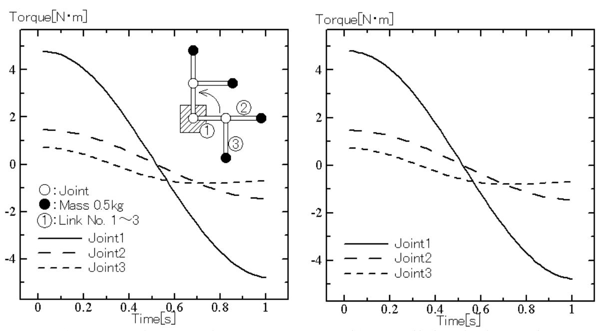


Figure 3 Flow chart of construction algorithm of member length matrix

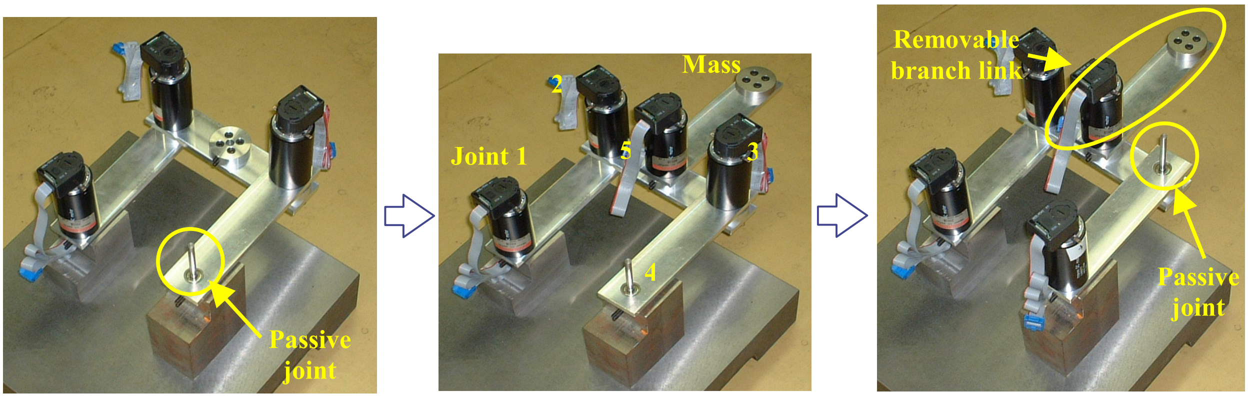
1) Starting from link No. n, place the member length matrix of the child link (link No. i) into the i -th row, i -th column of the total matrix.
2) Add all the components on the i -th row (k=1, ... , n) to the j -th row. If the mother link (link No. j) of the child link (link No. i) is the base, skip this step.
3) Repeat steps 1) and 2) until link No. 1 is reached.

By using this flow, various types of system configurations can be expressed explicitly and automatically. However, the member length matrix only expresses the structural connectivity of link systems; the



(a) Dynamic equations (b) Parallel solution scheme

Figure 4 Estimation of torque curves



(a) 4-joint closed-loop link system (b) 5-joint multibranch link system (c) 5-joint multibranch link system with passive joint at different position

Figure 5 Structure-varying link system

exchanges of forces at passive joints are handled in the vector $\{P^n\}$, if necessary. Examples showing structural connectivity of link systems are described in the next chapter.

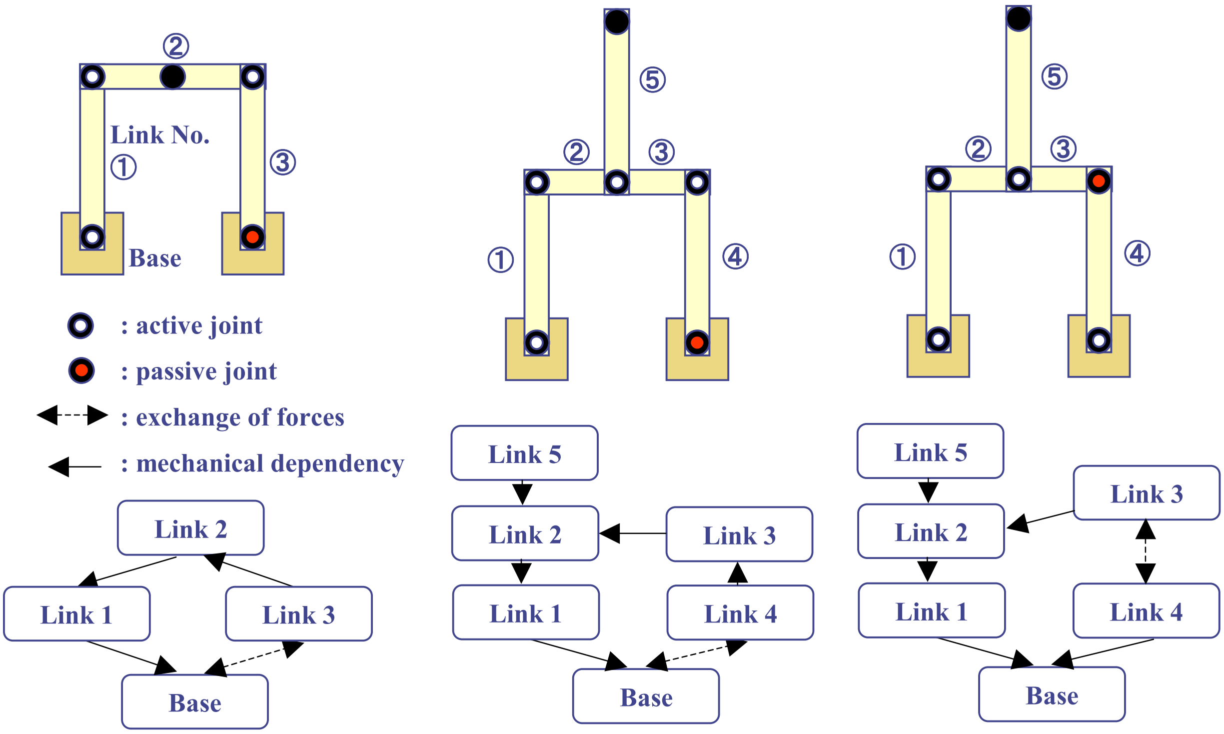
C. Numerical Example

In this section, the joint torque curves of a simple multibranch link system (length of each link: 0.4 m; link weight: 0.215 kg; center of gravity at midpoint; mass at the tip: 0.5 kg) are calculated and compared in two different ways; (a) dynamic equations derived by using the Lagrangian method, and (b) the parallel solution scheme. A 0.5 π rad rotational motion in 1.0 s is given in the horizontal plane. Fig. 4 shows the model and comparison of the obtained torque curves. As the branching joint is assumed to have two independent shafts (Joints 2 and 3), two torque values are obtained at the joint. As shown in the figure, the torque curves obtained by the two schemes are in good agreement with one another.

It is not impossible to derive the dynamic equations for more complex systems, although it is still difficult. However, the complexity can be dealt with only by revising the structural connectivity data for the parallel solution scheme.

III. Control Experiments on Multibranch Link Systems

Some control experiments are carried out on a structure-varying link system as shown in Fig. 5, in order



(a) 4-joint closed-loop link system (b) 5-joint multibranch link system (c) 5-joint multibranch link system with passive joint at different position

Figure 6 Structural connectivity of structure-varying link system

TABLE I. MOTOR DYNAMICS AND FEEDBACK GAIN

Parameters of motor dynamics		
J [kgm^2]	***D [kgm^2/s]***	***f_c [N]***
0.000384	0.00015	0.00015
Feedback gain values		
K_P [Nm/rad]	***K_I [Nm/rad]***	***K_D [Nms/rad]***
15.0	0.80	0.10

to verify the validity of torque values obtained by the parallel solution scheme, along with its flexibility against changes in structural connectivity. Link members are made of duralumin, with its length being 0.142 m, 0.180 m (branch link), and its weight being 0.102 kg, 0.130 kg (branch link). A mass at the tip of the branch link weighs 0.1 kg. Gearless motors (Maxon RE40, weight: 0.534 kg) are used in order to maximize the effect of the dynamics. The input torque for motors is calculated using

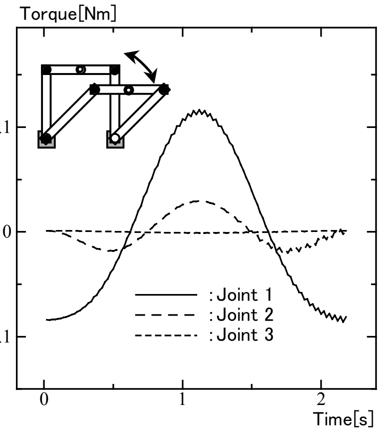
$$\tau = J\ddot{q}_d + D\dot{q}_d + f_c + \tau_{I.D.} + \tau_{feedback}, \tag{7}$$

where J is the moment of inertia of motors, D the viscosity coefficient of motors, f_c the dynamic friction force, $\tau_{I.D.}$ the input torque calculated by the parallel solution scheme, and $\dot{q}_d$ and $\ddot{q}_d$ the target angular velocity and angular acceleration, respectively. $\tau_{feedback}$ is the PID feedback torque, which is obtained using

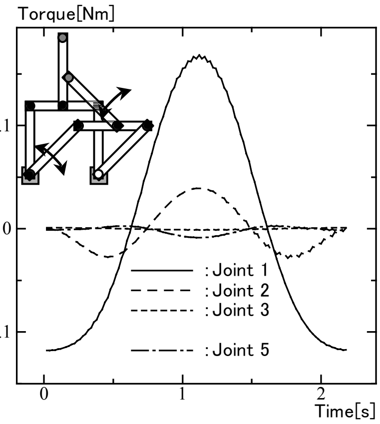
$$\tau_{feedback} = K_P(q_d - q) + K_I\Sigma(q_d - q) + K_D(\dot{q}_d - \dot{q}), \tag{8}$$

where q and $\dot{q}$ are the actual angle and angular velocity acquired from the attached encoders, respectively. K_P, K_I and K_D are the feedback gain for the angle, the integrated value and the angular velocity, respectively. These are fixed to specific values throughout the experiments in order to clarify only the effect of feed-forward torque. The parameters of motor dynamics and feedback gain are shown in Table 1.

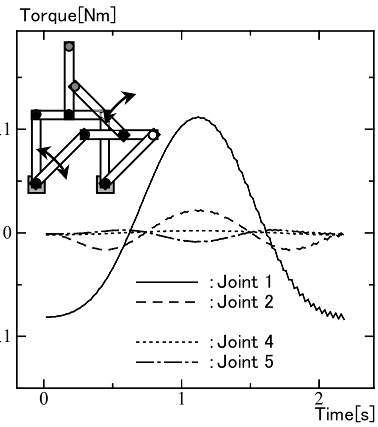
The link system is controlled, first as a 4-joint closed-loop link system, and then it is reconstructed to a 5-joint multibranch link system to carry out the control, and then to a system with its passive joint at a different position, to carry out the same routine. Fig. 6 shows the structural connectivity of the structure-varying link system, which is used in the construction of the member length matrix. A solid line and an arrow indicate the mechanical dependency between link members. For example, Link 3 and Link 5 are both dependent on (or supported by) Link 2 in Fig. 6(b), since joint torques of Links 3 and 5 are added to those of Link 2. A broken line and an arrow indicate the exchange of forces between the link members, which can be handled by the vector related to nodal forces, $\{P^n\}$. Fig. 7 shows the joint torque curves obtained by the parallel solution scheme, when a 2-second swaying motion as shown in the figure, is applied to each system configuration. Torque curves vary between each figure, due to the change of the dynamics of the system. In particular, we can confirm a reduction in maximum torque values when a passive joint is placed near the branching joint and the mass.



(a) 4-joint closed-loop link system



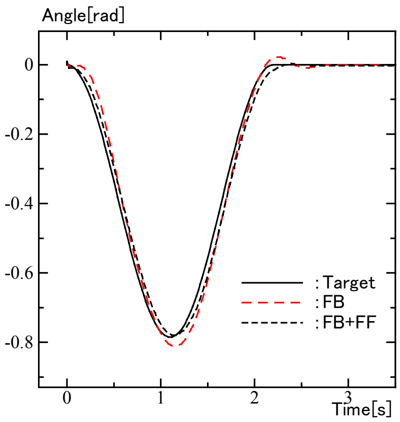
(b) 5-joint multibranch link system



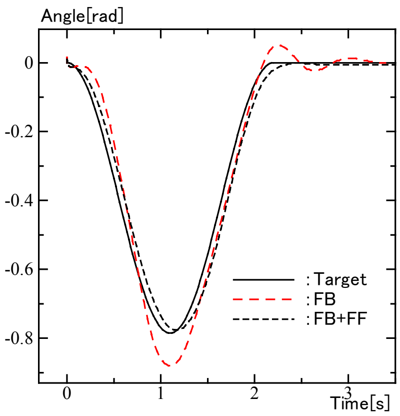
(c) 5-joint multibranch link system with passive joint at different position

Figure 7 Joint torque curves obtained by the parallel solution scheme

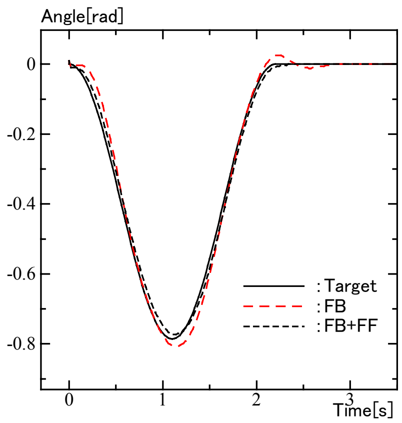
Fig. 8 shows the control results for Joint 1. It is evident that the feed-forward torque has a good effect on tracking against the target trajectory, compared to the results obtained only with feedback values. The performance is all consistently and similarly improved, though the dynamics of the link system is drastically changed. These results confirm the validity of the torque values calculated by the parallel solution scheme.



(a) 4-joint closed-loop link system



(b) 5-joint multibranch link system



(c) 5-joint multibranch link system with passive joint at different position

Figure 8 Control results for Joint 1

IV. CONLUDING REMARKS

A matrix-form equation for calculating torque values in the parallel solution scheme is separated into terms of different parameters. The structural connectivity of link systems, therefore, can be expressed explicitly by one of the matrices, the member length matrix. Its construction process is easy to implement in a program, and has high expansibility and flexibility. The solution scheme may demonstrate its flexibility in a fail-safe system, in such cases where a sudden structural change of link systems may occur.

One of the other terms in the matrix-form equation, the vector related to nodal forces, contains values with both static and dynamic effects. Studies on force control and inverse dynamics calculations for flexible manipulators, which can be developed by using this feature, are in progress.

ACKNOWLEDGMENT

The authors would like to extend their appreciation for the contributions of the former members of our laboratory, Takeo Ueda, Daisaku Imaizumi, Youichi Chikugo and Shunsuke Sato.

REFERENCES

[1] Y. Nakamura, “Dynamics computation of closed link robots and optimization of actuational redundancy,” Trans. Society of Instrument and Control Engineers, vol. 25, no. 5, 1989, pp.600–607, in Japanese.

[2] K. Sugimoto, “Deviation of equation of motion of closed loop mechanisms,” J. Robotics Society of Japan, vol. 15, no. 3, 1997, pp.460–467, in Japanese.

[3] Y. Nakamura and K. Yamane, “Dynamics computation of structure-varying kinematic chains and its application to human figures,” IEEE Trans. Robotics and Automation, vol. 16, no. 2, 2000, pp.124-134.

[4] D. Isobe, Y. Chikugo, D. Imaizumi, S. Sato and A. Yagi, “Feed-forward control of link mechanisms under various boundary conditions by using a parallel solution scheme,” Proc. 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2003), 2003, pp.2773-2778, Las Vegas U.S.A.

[5] D. Isobe and D. Imaizumi, “A parallel solution scheme of inverse dynamics for flexible manipulators,” Proc. IASTED International Conference on Robotics and Applications (RA2003), 2003, pp.106-111, Salzburg Austria.

[6] N.M. Newmark, “A method of computation for structural dynamics,” A.S.C.E. J. Engineering Mechanics, vol. 85, 1959, pp. 67-94.