A PARALLEL SOLUTION SCHEME OF INVERSE DYNAMICS FOR FLEXIBLE MANIPULATORS

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ABSTRACT

In this paper, we describe a parallel solution scheme of inverse dynamics, and its application to flexible manipulators where elastic deformation and vibration normally occur in constituting link members. The scheme is developed by using the Finite Element Method (FEM), which evaluates the analyzed model in absolute Cartesian coordinates, with the equation of motion expressed in dimension of force. The calculated nodal forces are converted into joint torques by using a matrix form equation divided into terms of force, transformation between coordinates, and length. Therefore, information from the entire system can be handled in parallel, which makes it seamless in application to any type of link system regardless of its stiffness value. The proposed scheme is combined with a kinematics solution scheme also developed by using the FEM, which conducts target trajectories for flexible manipulators. Numerical tests are carried out on simple link systems to verify the validity of the proposed scheme.

KEY WORDS
Flexible Manipulators, Inverse Dynamics, Kinematics, Parallel Solution Scheme, Finite Element Method

1. Introduction

Dynamic equations conducted by generally used schemes, such as the Newton-Euler method or the Lagrangian method, include interdependent variables between the constituting links, since they are evaluated in relative polar coordinates and in dimension of torque. Accordingly, it will become highly complicated to derive inverse dynamics of closed-loop link mechanisms, or of continuously transforming ones. In contrast, a parallel solution scheme, developed by using the Finite Element Method (FEM), evaluates the analyzed model in absolute Cartesian coordinates with the equation of motion expressed in dimension of force [1]. By taking advantage of natural characteristics of the FEM, i.e., the capability of expressing the behavior of each discrete element as well as that of the entire continuous system, local information such as nodal forces and displacements can be calculated in parallel. The nodal forces are calculated incrementally in a matrix form, which does not require any revision of the outside frame, and the variables inside can be revised by simply changing the input data in the case of a physical change in the hardware system. The calculated nodal forces are then converted into joint torques by using a matrix form equation divided into terms of force, transformation between coordinates, and length. The structure of the algorithm makes it seamless in application to different types of link mechanisms under various boundary conditions such as open- or closed-loop link mechanism [2,3]. The scheme can be applied to such circumstances where robotic tasks include motions that generate open and closed loops alternately.

On the other hand, the lightening of constituting members and devices is taking place in many robots, in order to increase their mobility. It may also increase safety among users, but it may well cause deficiency of structural strength of the architecture. Moreover, the elastic vibration that occurred in the link members becomes a serious issue for the control. Therefore, strong efforts have been taken to model [4-7] and calculate inverse dynamics [8] of robotic arms with elastic members. However, a large barrier before calculation of the inverse dynamics is possible, which comes from difficulties in handling the dynamic equations.

In this paper, we describe a three-dimensional version of the parallel solution scheme of inverse dynamics for link mechanisms. A kinematics solution scheme is also developed by using the FEM and combined with the parallel solution scheme, to enable consideration of flexural stiffness of the link members. We used the equation of motion based on the principle of virtual work considering both operation distance and deformation values in displacements. The calculated trajectories for flexible manipulators are applied as input for the parallel solution scheme of inverse dynamics. Numerical tests are carried out on several types of simple flexible manipulators, and the results are compared with those of rigid body models to verify the validity of the proposed scheme.

2. Parallel Solution Scheme of Inverse Dynamics

Figure 1 shows the nodal forces (based on global coordinates) acting on the i-th link (i=1~n) in a three-dimensional open-loop n-link mechanism. For simplicity, the distributed nodal forces acting along the link member are shown as concentrated force acting at the center of gravity. The joint torque τ_{ix} required around the x-elemental axis on the i-th link, for example, is determined by adding an i+1-th joint torque $\tau_{(i+1)x}$ to the sum of inertia moments acting on this link, and is expressed by nodal forces F_{iy} and $F_{i\phi x}$ based on elemental (or link) coordinates as follows:

$$\tau_{ix} = l_{iC}F_{iy} + l_i(\sum_{j=i+1}^{n} F_j)_y + F_{i\phi x} + \tau_{(i+1)x}\ , \tag{1}$$

where l_{iC} is the length between the former joint and the center of gravity and l_i is the link length. By considering other components around the y- and z-axes, and arranging them into global coordinates (X, Y, Z) in a matrix form, the joint torque vector is expressed as

$$\{\tau^n\} = [L^n][T^n]\{P^n\}, \tag{2}$$

where $\{P^n\}$ is a vector related to nodal force, defined as

$$\{P^n\} = \begin{Bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_n \end{Bmatrix},\ where\ \{P_i\} = \begin{Bmatrix} F_{iX} \\ F_{iY} \\ F_{iZ} \\ \sum_{j=i+1}^{n} F_{jX} \\ \sum_{j=i+1}^{n} F_{jY} \\ \sum_{j=i+1}^{n} F_{jZ} \\ F_{i\phi X} \\ F_{i\phi Y} \\ F_{i\phi Z} \end{Bmatrix}. \tag{3}$$

The transformation matrix $[T^n]$ is expressed as

$$[T^n] = [h^n][T^n_{GE}], \tag{4}$$

where $[h^n]$ is a correction matrix between x-y and z-x coordinate systems, which simply inverts their signs in the y-axis direction. $[T^n_{GE}]$ is a transformation matrix between global and elemental coordinates which is expressed as

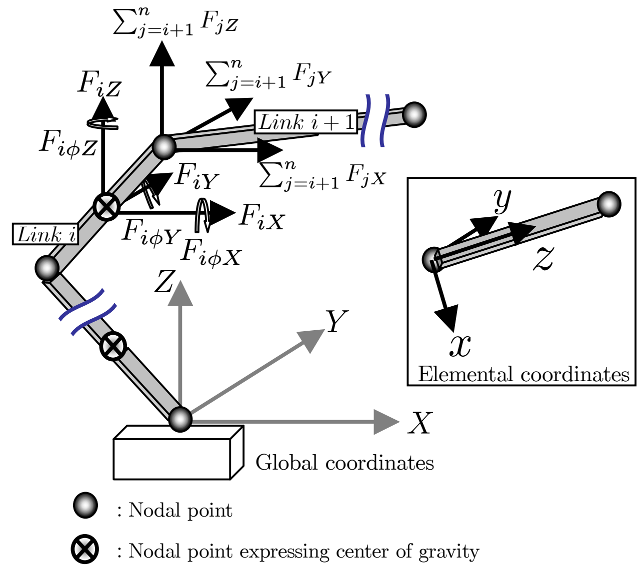


Fig. 1 Nodal forces acting on i -th link in an open-loop n-link mechanism

$$\begin{bmatrix} T^n_{GE} \end{bmatrix} = \begin{bmatrix} T_1 & & & & & \\ & T_2 & & & 0 & \\ & & T_3 & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & 0 & & & & \cdot \\ & & & & & T_n \end{bmatrix}, \tag{5}$$

where

$$\begin{bmatrix} T_i \end{bmatrix} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & A_i & 0 \\ 0 & 0 & A_i \end{bmatrix}, \tag{6a}$$

and

$$\begin{bmatrix} A_i \end{bmatrix} = \begin{bmatrix} cos\phi_{iXx} & cos\phi_{iYx} & cos\phi_{iZx} \\ cos\phi_{iXy} & cos\phi_{iYy} & cos\phi_{iZy} \\ cos\phi_{iXz} & cos\phi_{iYz} & cos\phi_{iZz} \end{bmatrix}, \tag{6b}$$

where ϕ_{iXx}, for example, represents the rotational angle between X-global and x-elemental coordinates. $[L^n]$ is a matrix related to member length and is expressed as

$$[L^n] = [T^n_\Lambda][\Lambda^n], \tag{7}$$

where $[T^n{}_\Lambda]$ is a transformation matrix between each elemental coordinate, and is expressed as

$$\left[\ T_\Lambda^n\ \right] = \begin{bmatrix} T_{11} & T_{12} & T_{13} & \cdot & \cdot & \cdot & T_{1n} \\ & T_{22} & T_{23} & \cdot & \cdot & \cdot & T_{2n} \\ & & T_{33} & \cdot & \cdot & \cdot & T_{3n} \\ & & & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot & \cdot \\ & 0 & & & & \cdot & \cdot \\ & & & & & & T_{nn} \end{bmatrix}. \tag{8}$$

$[T_{ij}]$ (i,j=1~n) is expressed using matrix $[A_i]$ shown above:

$$[T_{ij}] = [A_i][A_j]^T. \tag{9}$$

$[\Lambda^n]$ is expressed as

$$\left[\ \Lambda^n\ \right] = \begin{bmatrix} \Lambda_1 & & & & & & \\ & \Lambda_2 & & & 0 & & \\ & & \Lambda_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & 0 & & & & \cdot & \\ & & & & & & \Lambda_n \end{bmatrix}, \tag{10}$$

where

$$\left[\ \Lambda_i\ \right] = \begin{bmatrix} 0 & l_{iC} & 0 & 0 & l_i & 0 & 1 & 0 & 0 \\ l_{iC} & 0 & 0 & l_i & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

Information on the i+1~n link is summed by multiplying the $[L^n]$ matrix by vector $[T^n]\{P^n\}$, which is the nodal force vector transformed into elemental coordinates. In cases of closed-loop link mechanisms, the above matrix is divided into multiple parts, as shown below, to fix the configuration of passive joints as well as the torque allocation undertaken by active joints.

$$[L^n] = \begin{bmatrix} L^a & 0 \\ 0 & L^b \end{bmatrix} \tag{12}$$

The suffixes a and b are the numbers of links (a+b=n) when the mechanism is divided into two parts. This is the only process that is different between the algorithms of open- and closed-loop link mechanisms, which of course, can be automatically alternated in the program. A vector related to incremental nodal forces acting on the i-th link is defined using the nodal numbers k (=2i):

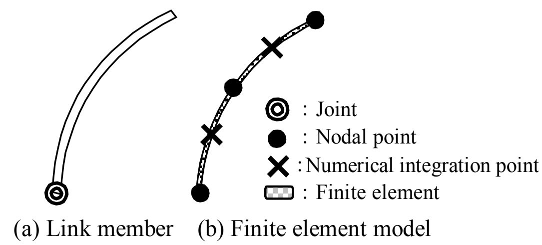


Fig. 2 Finite element modeling of a link member

$$\{\Delta p_k\} = \begin{Bmatrix} \Delta f_{kX} \\ \Delta f_{kY} \\ \Delta f_{kZ} \\ \sum_{h=k+1}^{2n+1} \Delta f_{hX} \\ \sum_{h=k+1}^{2n+1} \Delta f_{hY} \\ \sum_{h=k+1}^{2n+1} \Delta f_{hZ} \\ \Delta f_{k\phi X} \\ \Delta f_{k\phi Y} \\ \Delta f_{k\phi Z} \end{Bmatrix}. \tag{13}$$

Thus, the vector related to the nodal force acting on the i-th link at $t+\Delta t$ is successively calculated using the above vector as follows:

$$\{P_i\}_{t+\Delta t} = \{P_i\}_t + \{\Delta p_k\}. \tag{14}$$

The successive values of the n-link joint torque are then obtained by substituting Eq. (14) into Eq. (2).

A link mechanism constituted of a joint and a flexible link member, is modeled by using linear Timoshenko beam elements. Figure 2 shows an outline of the finite element modeling by using the beam elements. The element mass is divided equally between the two nodal points constituting the element. The numerical integration point for evaluating the stiffness of the element is fixed at the midpoint where accuracy against bending deformation is mathematically guaranteed in one-point integration. A more simple pin-joint rigid-bar link mechanism can also be expressed by summing the total mass of the elements at the nodal point expressing the center of gravity. The mass of a motor can be considered by placing the mass at the corresponding nodal points. Four-element subdivision per link is adopted in this study to maintain accuracy, and at the same time, to lower the calculation cost.

3. Solution Scheme of Kinematics for Flexible Manipulators

On calculating inverse dynamics for flexible manipulators, we need target trajectories that compensate for inertial forces acting at the link members, and also stiffness of the members. Therefore, a solution scheme of kinematics is also developed by using the FEM, and combined with the previously described solution scheme of inverse dynamics to handle analyzed models comprehensively in a single calculation process.

The incremental equation of motion at time $t+\Delta t$ conducted by the principle of virtual work is expressed as

$$[M]\{\ddot{u}\}_{t+\Delta t}+[C]\{\dot{u}_d\}_{t+\Delta t}+[K]\{\Delta u_d\}=\{F\}_{t+\Delta t}-\{R\}_t, \tag{15}$$

where $[M]$ is the total mass matrix, $[C]$ the total damping matrix, $[K]$ the total stiffness matrix, $\{F\}$ the external force vector and $\{R\}$ the internal force vector. $\{u_d\}$ is the displacement vector which contains only deformation values, and $\{u\}$ is the displacement vector which is the sum of both operation distance and deformation values, expressed as

$$\{u_d\}_{t+\Delta t}=\{u_d\}_t+\{\Delta u_d\}, \tag{16a}$$

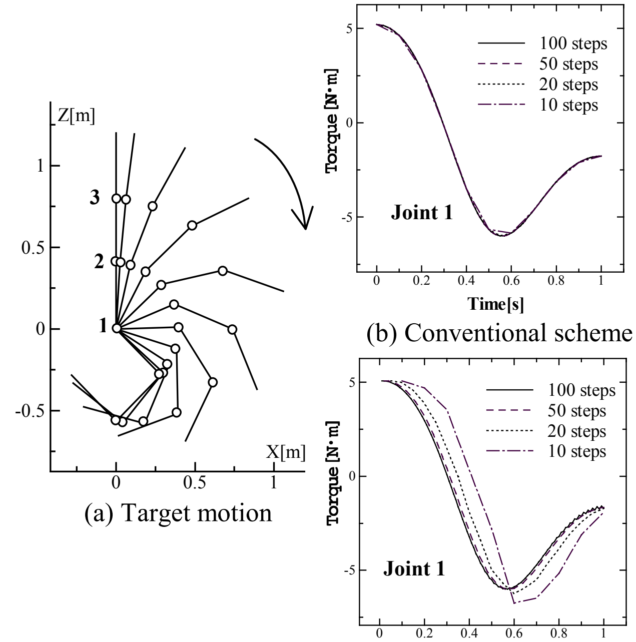
$$\{u_m\}_{t+\Delta t}=\{u_m\}_t+\{\Delta u_m\}, \tag{16b}$$

$$\{u\}_{t+\Delta t}=\{u_d\}_{t+\Delta t}+\{u_m\}_{t+\Delta t}, \tag{16c}$$

where $\{u_m\}$ is the displacement vector which contains only operation distance values. By applying $\{u_m\}$ as input in time integration loop of Eq. (15), we can successively obtain $\{u_d\}$ at each time step. Final target trajectories considering the effects of stiffness and damping are then obtained by using Eq. (16c). The resultant forces acting on the elements can also be calculated by using the obtained displacements. Newmark's β method (δ=1/2, β=1/4) is used as the time integration scheme to solve the incremental equation of motion.

4. Numerical Estimations

The parallel solution scheme for calculating inverse dynamics is applied to the joint torque calculation of an in-plane rigid-body link mechanism to examine the accuracy of torque curves against the number of incremental steps. Figure 3(a) shows the target trajectory for a 1.0s motion given in the vertical plane for a three-link mechanism (each link length: 40cm; each link weight: 215g; center of gravity at midpoint). Figures 3(b) and 3(c) show the torque curves obtained by using the conventional and proposed schemes. As shown in Fig. 3(b), the accuracy of the torque values does not depend upon the number of incremental steps when we use the dynamic equations that supply exact solutions. In contrast, the proposed scheme depends upon the number of incremental steps (see Fig. 3(c)), since the torque values are calculated approximately by summing the incremental information of each step. However, the results agree well when 50 to 100 steps per 1.0s of operating time, that is, less than 1 step per 10ms, are chosen.



(a) Target motion

(b) Conventional scheme

(c) Proposed scheme

Fig. 3 Accuracy of torque curves against number of steps

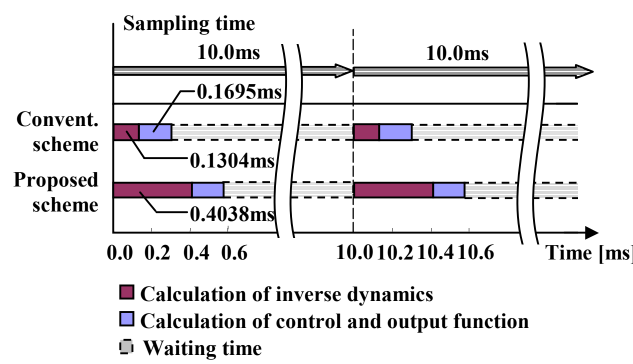
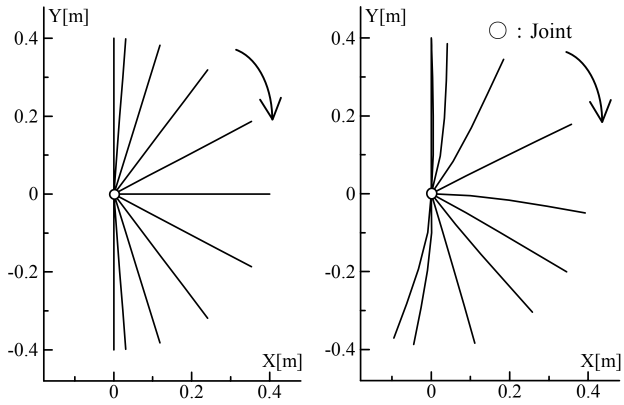


Fig. 4 Time possession of each process in control procedure

The time possession of each process in a control procedure is investigated to verify the sufficiency of the number of incremental steps against actual control (see Fig. 4). There are two main calculation processes during the control. One is the calculation of inverse dynamics, which may differ, of course, between the conventional and proposed schemes. The other is the calculation of control and output function, which is a process common to both schemes. The calculation time of inverse dynamics by using the proposed scheme is about three times longer than that by using the conventional dynamic equations. However, the process time is sufficiently short compared to the time of the entire process as shown in the figure, when, for example, the sampling time is selected to be a practical value of 10ms. Moreover, the accuracy of torque values is ensured by calculating only 1 step per sampling time, due to the estimation of the previous result.



(a) Rigid body model (b) Flexible model

Fig. 5 Kinematics of a 1-link manipulator

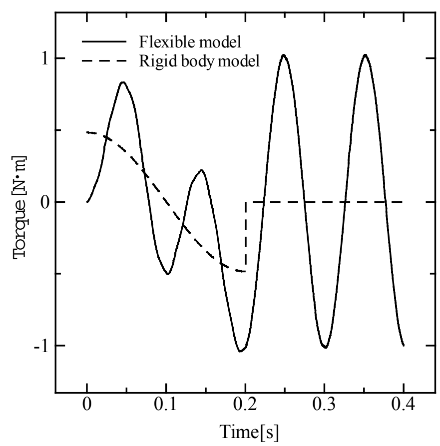


Fig. 6 Torque curves

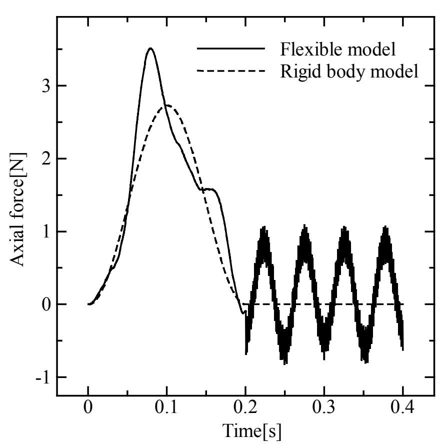


Fig. 7 Axial force

Estimations of calculation time were performed by a Pentium PC (CPU: 500MHz, memory: 512MB).

Although special attention must be given to the number of incremental steps, we can obtain torque curves for a closed-loop link mechanism or even those for a continuously transforming mechanism without revising any part of the numerical algorithm in the solution scheme [2,3]. This is one of the biggest merits of using the proposed scheme.

5. Numerical Examples

Numerical tests are carried out on flexible manipulators. Damping is neglected in the calculations. Figure 5(a) shows a target trajectory of a 0.2s motion given in the horizontal plane for a rigid body 1-link system (link length: 40cm; weight: 22.4g; flexural stiffness: perfect rigidity). By using this trajectory as input for the kinematics solution scheme described above, a trajectory considering the effect of stiffness is obtained as shown in Fig. 5(b). The flexural stiffness for the flexible model is given as 0.46Nm. From the results, we can see the difference between the deformations of the two link systems. Figure 6 shows the joint torque curves for the two models obtained by the inverse dynamics solution scheme, using the trajectories of Figs. 5(a) and 5(b) as input data. We can see that a smooth joint torque curve is obtained for the rigid body model. In contrast, a vibration can be observed in the torque curve of the flexible model due to the flexural vibration that occurred in the model. A similar phenomenon appears in the time histories of axial force (see Fig. 7), where the time history for the flexible model vibrates especially after the motion has stopped. The estimated natural frequency of the model is in good agreement with the theoretical value, which is 10.02Hz.

Another numerical test for a 2-link system is carried out. A different stiffness value for each link member is adopted to see the influence of the parameter in such cases. Parameters for the model are as follows: each link length: 20cm, each link weight: 11.2g, and flexural stiffness: 0.46Nm (Link 1) and 0.046Nm (Link 2). Joint 1 is rotated for 0.5rad, and Joint 2 is rotated for 1.0rad in 0.2s. As we can see in Fig. 8, Link 2 is much more

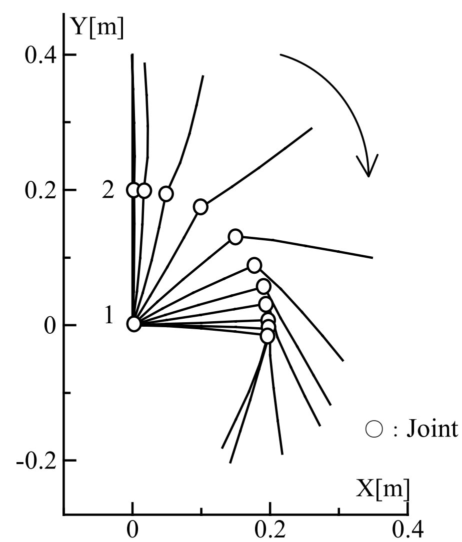


Fig. 8 Target trajectory for a 2-link manipulator with different stiffness values for each link

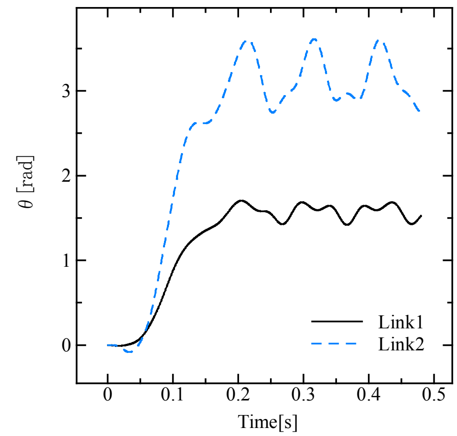


Fig. 9 Target trajectory (rotational angle)

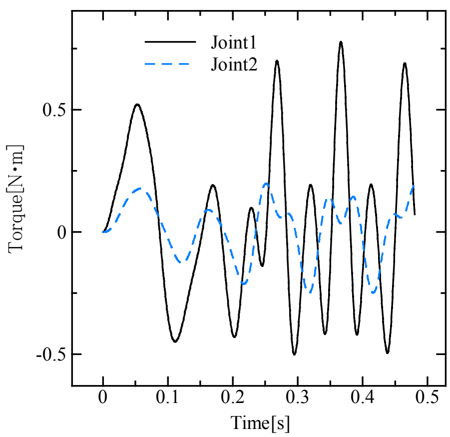


Fig. 10 Torque curves

deformed than Link 1 due to the lower stiffness value. Moreover, a combination of various vibration modes can be observed in both the target trajectory (Fig. 9) and the torque curves (Fig. 10). This comes from the difference of natural frequencies of each link. The results lead us to conclude that the solution scheme of kinematics developed by using FEM can handle the systems with links of various stiffness values. Also, it is confirmed that the parallel solution scheme of inverse dynamics can be commonly applied, and without any revision, to both rigid body models and flexible models.

6. Conclusion

A parallel solution scheme of inverse dynamics is applied to flexible manipulators, where elastic deformation and vibration normally occur in constituting link members. It derives nodal forces in parallel and converts them to the joint torque, which can commonly be applied to any type of link mechanism regardless of its stiffness value. No revision of the basic numerical algorithm is required regardless of the type of model we use. This function cannot be realized by using the conventional schemes based upon the generally used dynamic equations. The scheme is also valid for link mechanisms under various boundary conditions, which may help us to achieve stability and smoothness in continuous motions of complex robotic architecture with elastic members. Control experiments on flexible manipulators using the proposed scheme are scheduled.

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