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RISK ESTIMATION FOR PROGRESSIVE COLLAPSE OF BUILDINGS

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Abstract. *In this study, the collapse behaviours of steel-framed buildings were simulated using the adaptively shifted integration (ASI) -Gauss code to investigate the relation between a key element index, which indicates the contribution of a structural column to the vertical capacity of the structure, and the scale of progressive collapse. Collapses were initiated by removing specific columns from models designed based on different axial force ratios, and the collapse scales of the buildings were estimated using the total potential energy values of structural members after collapse. The numerical results for various models with different structural strengths confirmed that a larger integrated value for the key element index resulted in a higher risk of progressive collapse. This index could be used to predict and compare the progressive collapse risks for a building even when various locations for the removed columns are assumed.*

1 INTRODUCTION

In the case of an emergency such as an intense fire or the collision of an object with a building, there is a risk of progressive collapse. This was the phenomenon that occurred at the World Trade Center (WTC) towers during the 9.11 terrorist attacks. The official statements released by the Federal Emergency Management Agency (FEMA) in 2002[1] and the National Institute of Standards and Technology (NIST) in 2005[2] and 2008[3] concluded that the details of the failure process after the decisive initial trigger that set the upper part in motion were very complicated and their clarification would require large computer simulations. Among the various papers published after the incident, the one published by Bazant et al.[4] has become a milestone for numerous other works concerning the progressive collapse of buildings. By defining an overload ratio, the ratio of the elastic strength of the lower structure to the total weight of the upper, falling structure, they explained that the force applied to the lower structure by the upper overwhelmingly exceeded the design vertical load, and consequently, it would be practically impossible to design a high-rise tower to avoid a progressive collapse such as that which occurred in the case of the twin towers. On the other hand, they also admitted that numerous computer simulations should be carried out to clarify the collapse process, because their estimations were performed without considering the tilting of the upper structure, member fracture, and other details.

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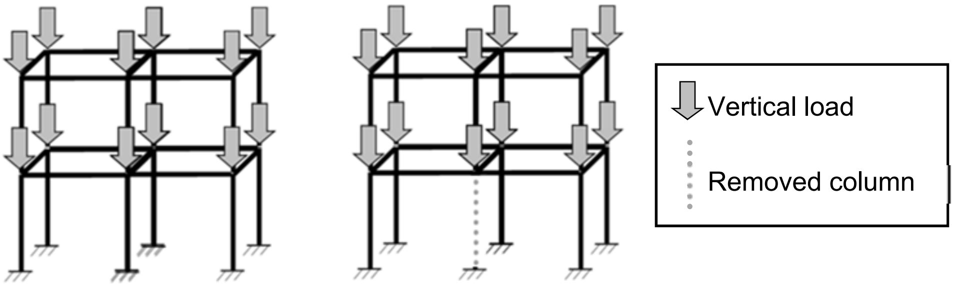


Figure 1: Pushdown procedure for calculating key element index

In this study, the collapse behaviours of steel-framed buildings were simulated using the adaptively shifted integration (ASI) -Gauss code to investigate the relation between a key element index that indicates the contribution of a structural column to the vertical capacity of the structure and the scale of a progressive collapse. Collapses were initiated by removing specific columns from models designed based on different axial force ratios. Various patterns of removed columns, of which the locations were restricted to a single floor, were investigated. The total potential energy values of the structural members after the collapse were used to estimate the collapse scales of the buildings.

2 KEY ELEMENT INDEX

Because of differences in their span lengths and strengths, certain columns in a structure support greater vertical loads than other columns and thus act as "key elements" in the structure. Several indexes have been developed to evaluate the contribution of each column to the strength of a structure, i.e., to determine the key elements, such as the redundancy index[5] and sensitivity index[6]. These indexes are effective at identifying highly sensitive columns that support the vertical load; however, they consistently identify those columns that would only cause a partial collapse at the upper floors and thus they are not useful for estimating the scale of a progressive collapse. Therefore, a new index called the key element index was proposed for estimating the contribution of base columns to the total collapse of a structure[7].

The key element index is calculated as follows. First, a static pushdown analysis is performed, as shown in Figure 1, by incrementally applying vertical loads at every structural joint in a modelled structure. Even though several columns on the upper floors may yield in this process, the vertical loads continue to be applied in steps until one of the base columns yields. The total vertical load (including the floor loads) applied to a column at the step where one of the base columns yields is defined as the ultimate yield strength P_G of the structure. The ultimate yield strength of the initial, undamaged structure is denoted as ${}_0P_G$, and the ultimate yield strength of a structure with one base column eliminated is denoted as ${}_1P_G$. The key element index KI of column m, for example, can be defined as the ratio of the ultimate yield strengths of the present and initial steps. The index can thus be written as follows:

$$ {}^{0}_{1}KI_m = {}_0P_G / {}_1P_G \tag{1} $$

The key element index at the nth step is then written as follows:

$$ {}^{0}_{n}KI_m = {}_0P_G / {}_nP_G \tag{2} $$

The index shown above indicates the contribution of the present column (or columns) to the strength of the initial, undamaged structure and can be used for cases in which the columns are removed simultaneously. However, the sensitivities of columns may change momentarily in cases where the columns are removed sequentially within a certain interval (such as blast demolition cases). The updated index, as shown below, is used for those cases with a sequential removal:

$$ {}^{n-1}_{n}KI_m = {}_{n-1}P_G / {}_nP_G \tag{3} $$

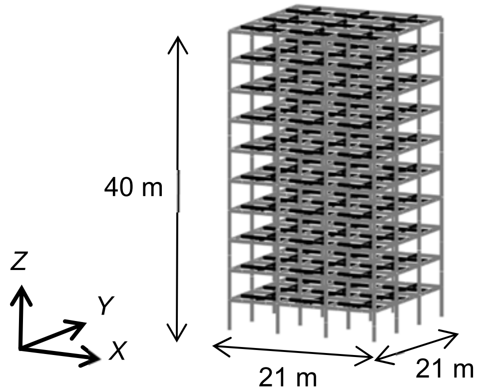


Figure 2: Numerical model

In this case, the index can be defined as the ratio of the ultimate yield strengths of the present and previous steps. An example of the distribution of key element index values in a building model can be found later in this paper.

3 NUMERICAL MODELS AND CONDITIONS

A ten-story, three-span steel framed building model, as shown in Figure 2, was constructed for the progressive collapse analysis. Each span length of the model was 7 m, and the height between floors was 4 m. The columns had box-type sections and were made of SM490 steel; the beams had H-type sections and were made of SM400. The material properties are listed in Table 1. The columns were subdivided with two linear Timoshenko beam elements per member, and the beams and floors were subdivided with four elements. The floor elements were all modelled with elastic elements that did not yield. Several models with different strengths were prepared, as listed in Table 2, to determine the effects of various structural strengths on the collapse scale of a building. A floor load of 800 kgf/m^2 was considered, and the sectional sizes of the columns and beams were based on the base shear coefficients of buildings when designing the models. These models are referred to as models A, B, C, D, and E, hereafter, from the strongest to the weakest.

The collapse modes of buildings were expected to change according to the number of removed columns and their locations. Thus, the patterns of removed columns were selected as shown in Figure

Table 1: Material properties of steel members

	E	σ_y	ν	ρ
SM400	206	245	0.3	7.9×10^{-6}
SM490	206	325	0.3	7.9×10^{-6}

E : Young's modulus [GPa], σ_y : Yield stress [MPa],

ν : Poisson's ratio, ρ : Density [kg/mm^3]

Table 2: Structural strengths of each model

	Maximum axial ratio n	Base shear coefficient C_b
Model A	0.124	0.200
Model B	0.200	0.095
Model C	0.300	0.048
Model D	0.400	0.027
Model E	0.500	0.016

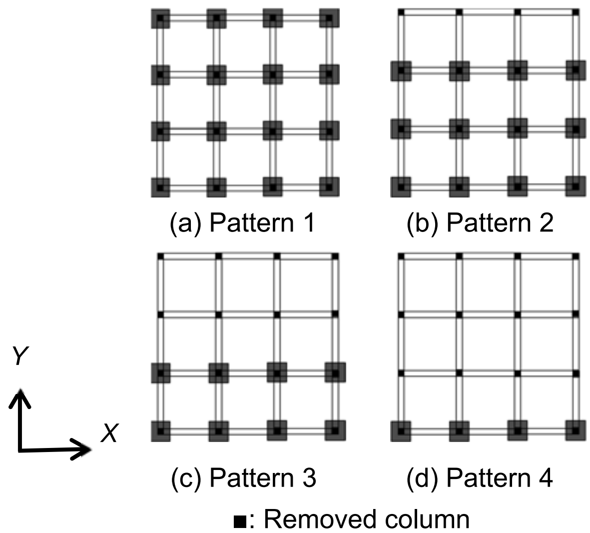


Figure 3: Removed-column patterns

3, and the removal of columns was restricted to a single floor for simplicity. An analysis was carried out for 10.0 s of simulation time with a time increment of 1.0 ms. The removal of columns was executed at t = 1.0 s in the simulations.

A potential energy decrease ratio, as defined in the following equation, was used to evaluate the scale of the progressive collapse:

$$\text{Potential energy decrease ratio} = \frac{U_0 - U_f}{U_0} \tag{4}$$

where U_0 and U_f are the potential energies of a numerical model at the initial and final states, respectively. The potential energy U is defined as the sum of the potential energies of all the elements composing the model and is expressed as follows:

$$U = \sum_{i=1}^{i_M} mgh_i \tag{5}$$

where i, i_M, m, g, and h_i are the element number, number of elements besides fractured ones, mass of the element, gravitational acceleration, and height from the ground to the centre of element no. i, respectively. The scale of the progressive collapse is overestimated. Thus, it can be underestimated from a safety point of view by only applying Eq. (5) to non-fractured elements. The value calculated by Eq. (4) indicates a larger scale of progressive collapse if the value is closer to 1.0.

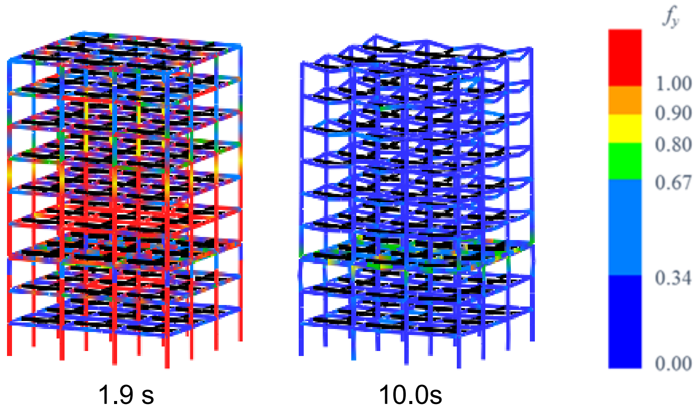
4 PROGRESSIVE COLLAPSE BEHAVIOURS OF STEEL-FRAMED BUILDING

As mentioned in the previous section, the patterns shown in Figure 3 were used to investigate the collapse behaviours of the models. A total of 200 simulations were performed by varying the floor of the removed columns from the 1st floor to the 10th floor.

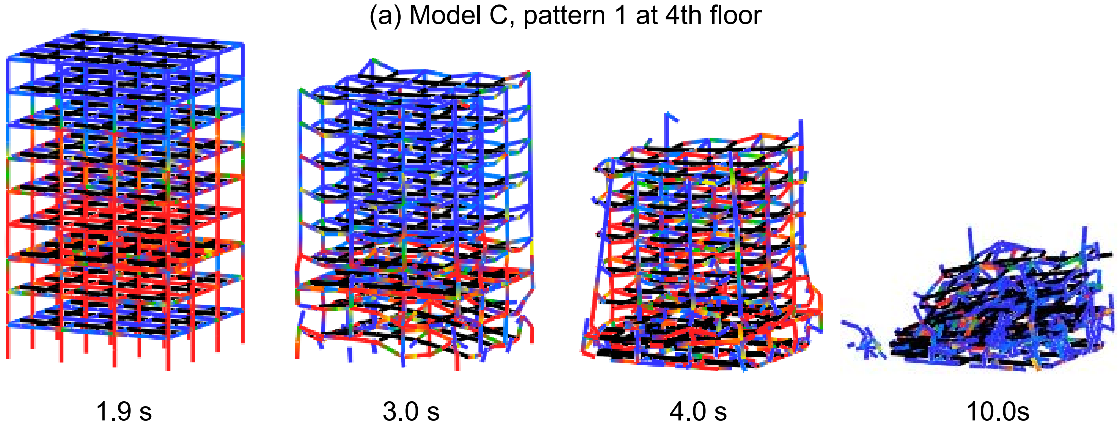
The collapse sequences of the models are shown in Figure 4. The colours in the figures show the yield function values, which are expressed as follows:

$$f_y = \left(\frac{M_x}{M_{x0}}\right)^2 + \left(\frac{M_y}{M_{y0}}\right)^2 + \left(\frac{N}{N_0}\right)^2 \tag{6}$$

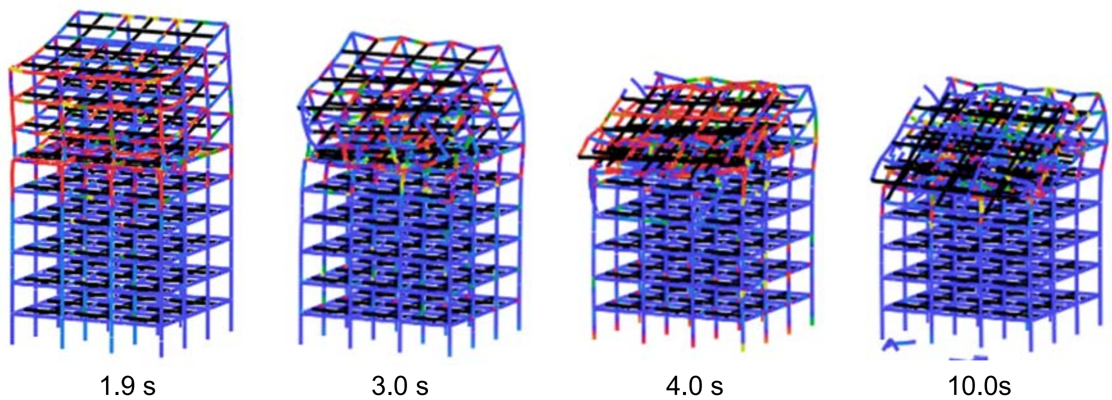
where M_x, M_y, and N are the bending moments around the x axis and y axis and the axial force, respectively. The terms with the subscript 0 are values that result in a fully plastic section in an element if they act on the cross section independently. The red colour, for example, shows that the yield function of the element has reached to a value of 1.0 and has yielded. First, Figures 4(a) and 4(b) show the results for models C and E when columns are removed with pattern 1 at the 4th floor, respectively. Shock waves propagating throughout the building are observed after the collision between the upper and lower structures occurs at t = 1.9 s. Then, model C, which has a relatively



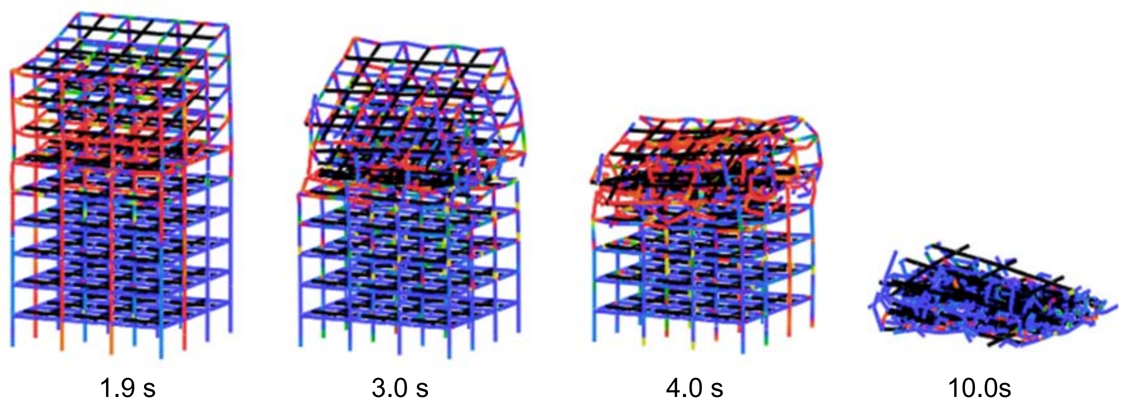
(a) Model C, pattern 1 at 4th floor



(b) Model E, pattern 1 at 4th floor

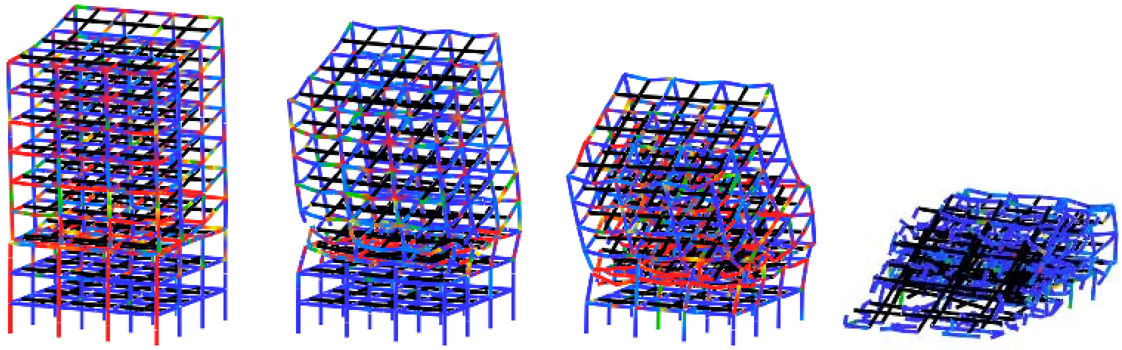


(c) Model C, pattern 2 at 7th floor



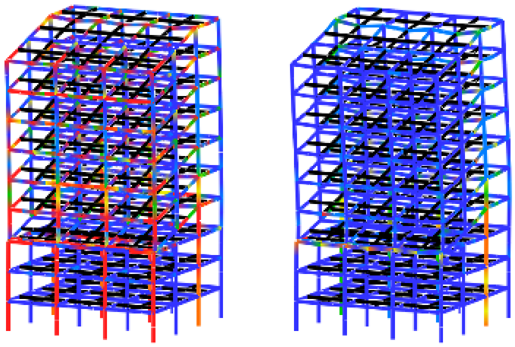
(d) Model E, pattern 2 at 7th floor

Figure 4: Collapse sequences under various conditions (continued)



1.9 s 3.0 s 4.0 s 10.0s

(e) Model C, pattern 2 at 4th floor



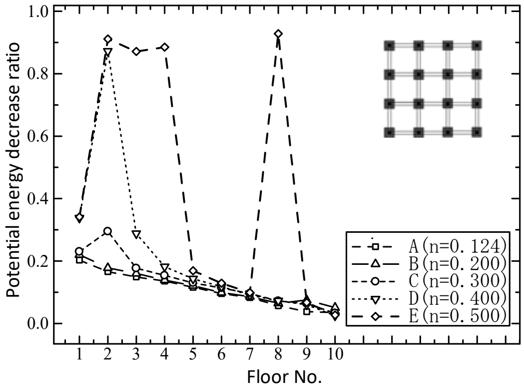
2.0 s 10.0 s

(f) Model C, pattern 3 at 4th floor

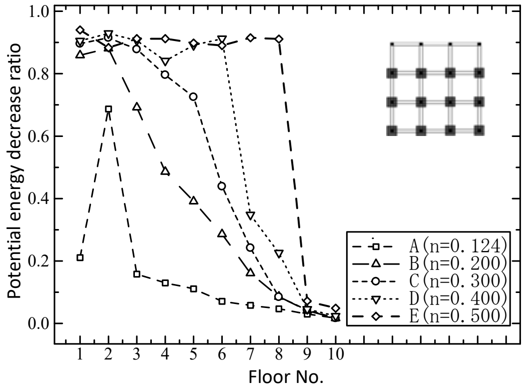
Figure 4: Collapse sequences under various conditions

strong structural strength, withstands the collapse and stops it from progressing. On the other hand, model E, which has a relatively weak structural strength, cannot withstand the impact, and finally advances to a total collapse. Asymmetricity appears when columns are removed with pattern 2 at the 7th floor for the same models (Figures 4(c) and 4(d)). Although the upper structure in model C crashes after the impact, it does not advance to a progressive collapse. In contrast, the tilting of the upper structure causes severe damage to the lower structure on impact in model E and triggers a total collapse. Figures 4(e) and 4(f) show the cases where columns are removed with patterns 2 and 3 at the same 4th floor for model C. Here, it is confirmed that the collapse is stopped from progressing when the number of removed columns decreases from pattern 2 to pattern 3. However, the risk of progression does not necessarily increase in accordance with the number of removed columns; the collapse is stopped from progressing in Figure 4(a) when all 16 columns are removed. The asymmetricity of the removed-column patterns might have relevance. Furthermore, it can be clearly confirmed by comparing Figures 4(c) and 4(e) that the risk of progression becomes higher if the removed columns are located on lower floors.

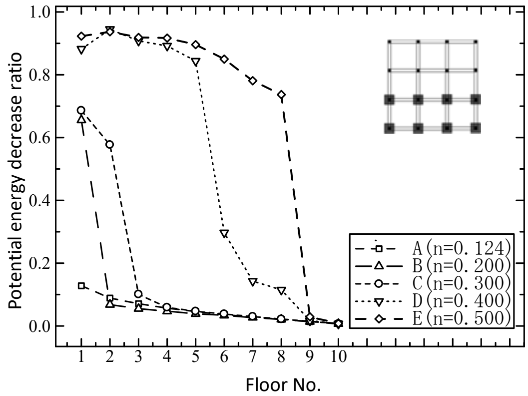
The numerical results of 200 cases indicated that the collapse modes tended to vary depending on the structural strength of the model, floor of the removed columns, and removed-column pattern. Figure 5 shows the relation between the floor number where the columns are removed and the potential energy decrease ratio for each pattern. The decrease ratio shows a clear dependency on the location of the floor where the columns are removed in patterns 2, 3, and 4 (Figures 5(b), 5(c), and 5(d)), with a lower floor showing a higher ratio. This indicates a high risk of large-scale collapse when the failure of columns occurs at lower floors. This is because the impact force of the upper structure on the lower structure increases when the upper has a larger kinetic energy. It is also a result of the tendency for a building to become more unstable if the columns at lower floors are removed. The figure also shows that the decrease ratio becomes larger as the strength of the model becomes weaker from model A to model E, and as the number of removed columns becomes larger. However, there is a slight difference in this tendency in the case of pattern 1, as shown in Figure 5(a). Although the number of removed columns is larger than in pattern 2, the removed-column



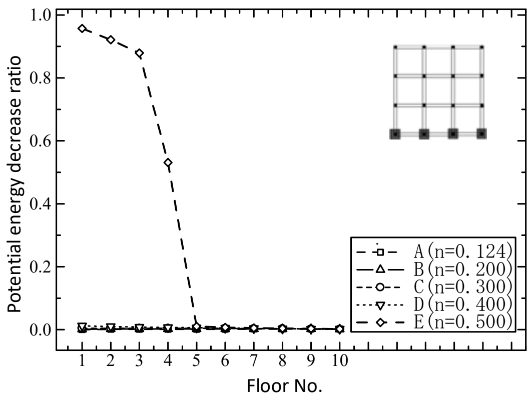
(a) Pattern 1



(b) Pattern 2

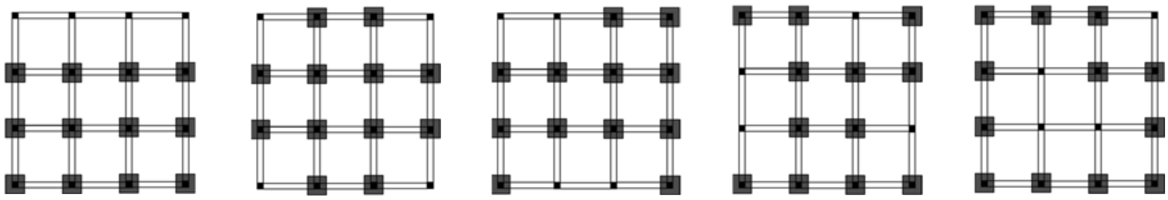


(c) Pattern 3

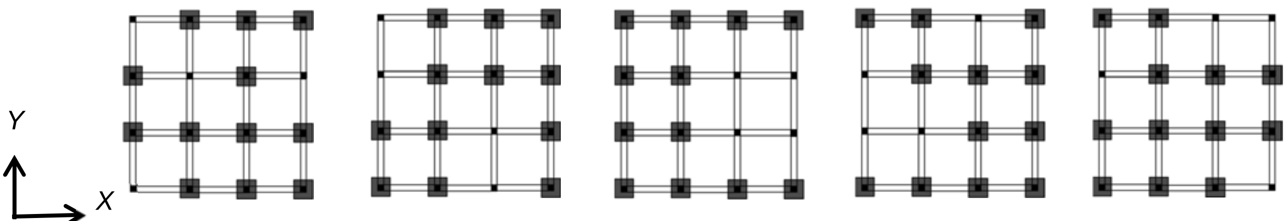


(d) Pattern 4

Figure 5: Relation between floor number where columns are removed and potential energy decrease ratio for each pattern



(a) Pattern 2-1 (b) Pattern 2-2 (c) Pattern 2-3 (d) Pattern 2-4 (e) Pattern 2-5



(f) Pattern 2-6 (g) Pattern 2-7 (h) Pattern 2-8 (i) Pattern 2-9 (j) Pattern 2-10

■: Removed column

Figure 6: Removed-column patterns for detailed investigation

pattern is fully symmetric and stable cases that do not lead to a large-scale collapse tend to appear more often. The impact between the upper and lower structures occurs in a wider area with less tilting in this case, and the impact force is distributed in a wider range compared to the other patterns, which should produce the stability.

5 RISK ESTIMATION FOR PROGRESSIVE COLLAPSE USING KEY ELEMENT INDEX

Next, by fixing the number of removed columns at 12, the removed-column patterns shown in Figure 6 were selected to investigate the relation between the integrated values of *KI* and the risk of progressive collapse. Pattern 2-1 in the figure is the same as pattern 2 used in the previous section, and it should be the most asymmetric pattern in Figure 6. Pattern 2-2, on the other hand, should be the most symmetric pattern of all. Other patterns were selected randomly. The asymmetricity of each pattern was evaluated by calculating the moving distance of the centre of gravity in the horizontal plane before and after the removal of columns, and the effect on the collapse scale was also investigated.

Figure 7 shows the key element index values calculated for each column in model A, as an example. The columns with larger *KI* values are illustrated with deeper grey in the figure. As a whole, the columns in the corner have smaller values and the ones at the lower floors have larger values. Other models also had the same tendencies. The integrated values of *KI* are calculated by summing up all the *KI* values for the removed columns.

Figure 8 shows the relation between the floor number where the columns are removed and the potential energy decrease ratio for models A, C, and E. The tendencies of the plots are the same as in the previous section, with a lower model strength and lower floor where the columns are removed leading to a higher ratio or risk of large-scale collapse. If the results are surveyed for each model, a variation in the ratio depending on the removed-column patterns can be observed. In

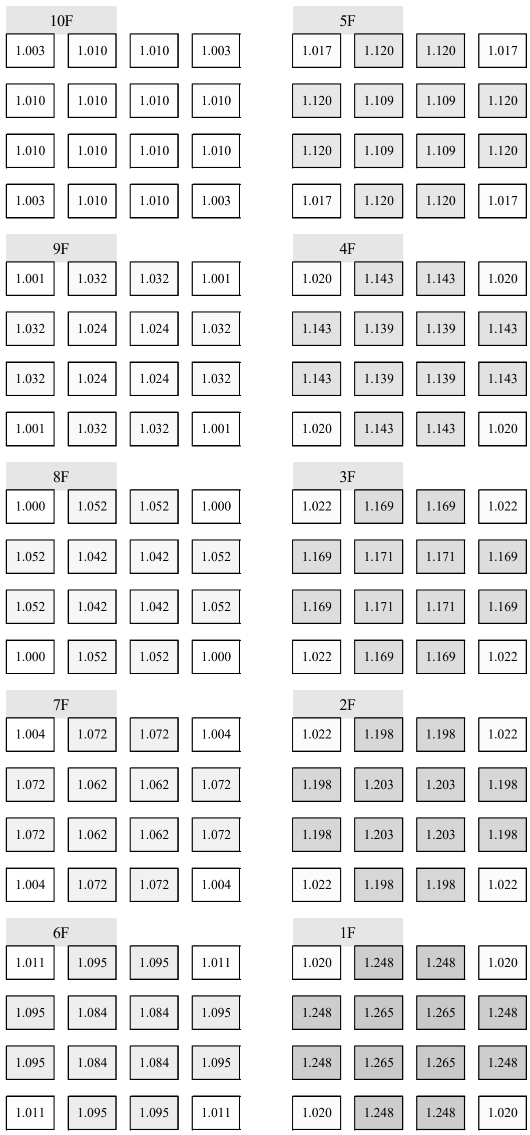
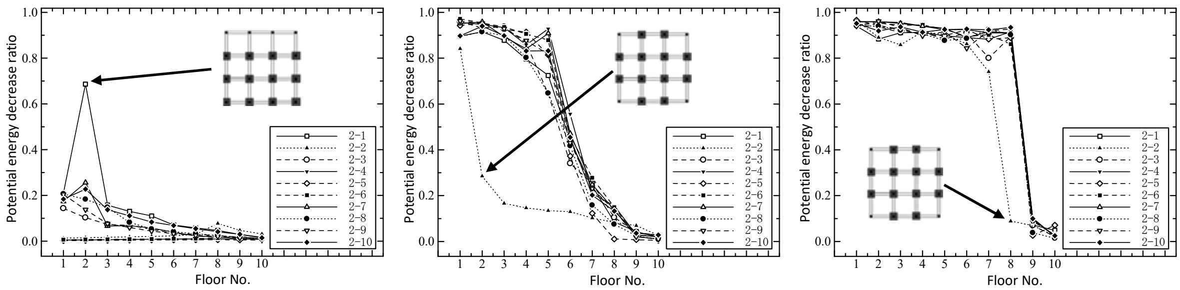
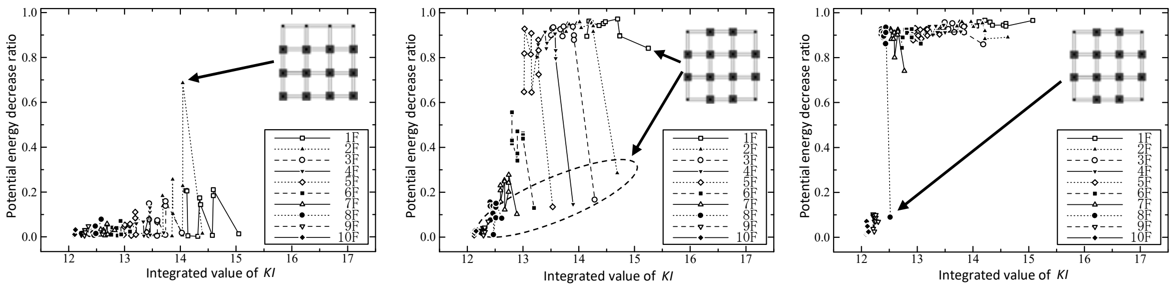


Figure 7: Key element index value of each column for model A



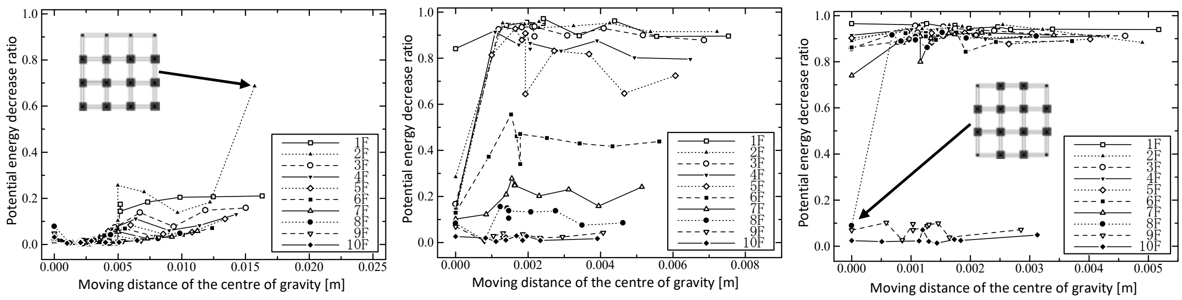
(a) Model A (b) Model C (c) Model E

Figure 8: Relation between floor number where columns are removed and potential energy decrease ratio for each model



(a) Model A (b) Model C (c) Model E

Figure 9: Relation between integrated values of *KI* and potential energy decrease ratio for each model



(a) Model A (b) Model C (c) Model E

Figure 10: Relation between moving distance of centre of gravity and potential energy decrease ratio for each model

particular, a large-scale collapse occurred only in case of pattern 2-1, the most asymmetric pattern of all, in model A, the strongest model of all. In contrast, the collapse scale became smaller in the case of pattern 2-2, the most symmetric pattern of all, in models C and E, the weaker models.

Figure 9 shows the relation between the integrated values of *KI* and the potential energy decrease ratio for models A, C, and E. It is evident that as the integrated value of *KI* becomes larger, the ratio, and thus the collapse scale, becomes larger. Moreover, the critical integrated value of *KI* where large-scale collapse begins becomes smaller as the strength of the model becomes weaker from model A to model E. This means that there is some critical integrated value of *KI* to start a large-scale collapse, which depends on the strength of the models. The conditions of pattern 2-1 on the 2nd floor of model A and pattern 2-2 in some cases of models C and E also show some irregular tendencies, as observed in Figure 8.

To see the effect of the asymmetricity of the building before and after the removal of columns, the relation between the moving distance of the centre of gravity in the horizontal plane and the potential energy decrease ratio was investigated, as shown in Figure 10. The mass of the removed columns is very small compared to that of the whole building, and the moving distance due to the removal is trivial. However, it can be observed that the collapse scale becomes larger in accordance with the moving distance in the horizontal plane. It should also be noted that the moving distances of the centre of gravity for patterns 2-1 and 2-2 are located at extreme positions in the figures, which explains the results shown in Figures 8 and 9. The collapse scale of a building is greatly affected by the asymmetricity or symmetricity of the structural failure.

6 CONCLUSIONS

By evaluating the numerical results using the key element index, it was found that a larger integrated value for the key element index results in a higher risk of progressive collapse. However, some peculiar tendencies were observed in the cases of removed columns with extremely symmetrical or asymmetrical locations. The critical integrated value of the key element index needed to cause a large-scale progressive collapse tends to depend on the strength of the building, and the value cannot be generally determined. This is because the key element index is a parameter that is not related to the strength of the building itself, and the relative values for buildings with different strengths cannot be compared. Therefore, the key element index may only be used to predict and compare the risks of progressive collapse in the same building, but this may be done even when various locations for the removed columns are assumed.

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